Monocular Compensation for Colour Deficient People

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Monocular Compensation for Colour Deficient People

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Colour deficient people, comprising 8% of the world population, are disturbed by their degraded colour discrimination power. This thesis discusses how their situation on visual display units (VDUs) can be improved with a presentation strategy called monocular compensation. By presenting a compensated view to one eye, missed colour discriminations can be redeemed. Original discriminations are guaranteed with an original view presented to the other eye at the same time. The additional hardware requirement is the stereoscopic display capability of the VDU.

However, such a presentation strategy inevitably introduces unfamiliar binocular dissimilarity. To minimize the associated visual discomfort, gamut-based palette compression (GBPC) has been developed for compensated view generation. GBPC is so named because the compensated view so produced essentially displays the original view with a compressed palette, whose entries are determined by a morphing of the VDU colour display gamut. The algorithm has the compensation level c as its operating parameter. Users can determine their own c value according to their binocular dissimilarity tolerance and discrimination improvement needed.

Images reproduced with VDUs have dominated the human visual world. We thus expect our overall design, named Stereo Visual Display Unit - Monocular Compensation (SVDU-MC for short), can become an effective and useful colour vision aid for the colour-blind.

摘要

佔世界人口百分之八的的色盲色弱人士，都在被自身較常人弱的辨色能力困擾。本論文探討如何運用「單目補償」技術，去改善他們在顯示器上的表現。單目補償乃透過展示於某一隻眼的補償映像，補回原先因色盲色弱而失去了的辨別資訊；而原先的辨別資訊，則透過展示在另一隻眼的原映像而得以保證。顯示器只需配備立體顯示功能，就能使用這個技術。

可是，這無可避免地要雙眼接收一組異常的相異映像。為了減低隨之而起的視覺不適，我們設計了「色彩範圍為本的色板壓縮」（ＧＢＰＣ）這個用作製作補償映像的演算方法。ＧＢＰＣ之所以如此命名，沿於其運用一壓縮了的色板來顯示補償映像的本質；至於色板如何壓縮，則取決於顯示器色彩顯示範圍的變型。該演算方法以補償程度ｃ作為其運作參數。用者可以根據自身對相異景象的接受程度，以及對補償的需要，來決定該參數的數值。

運用顯示器再生的映像，已經充斥於人類的視覺世界。故此我們預期，這個整體喚作「立體顯示器——單目補償」（ＳＶＤＵ－ＭＣ）的設計，能夠成為一個既有效又實用的辨色輔助工具。

Acknowledgement

First of all, I would like to express my gratitude towards my supervisor Professor NG Wai-Yin. He has inserted a lot of effort to this thesis, from topic brainstorming and problem break-down, to thesis organization and writing. I believe that my experience working with him, though being the harshest I have ever met, should at the same time be the most inspirational to be forgotten.

Special credits should also be given to Professor H. Swarbrick, Professor G. Meyer and Mr. S. Yang. They have been patient in answering my email questions concerning their published papers. Their replies have really helped me a lot in understanding the topic issue.

The time that my subjects have spent on my experiments should also be mentioned. Here I would like to specially thank two of them, SK and WH. Their first-hand comments have contributed to the quality experiments that you all see in this thesis.

What should not be missed are my family members, my friends, my teachers and my students. My study life would not have been so fun and joyful if I had not met and interacted with them.

Last but not least, I would like to thank the university libraries in Hong Kong. Their rich collections of books and journals have offered me many fruitful afternoons in these years of study.

This work is dedicated to Hong Kong,

where I was born and raised.

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Chapter 1

Introduction

Imagine you are now in a dark room. How would you pick a suitable pair of colour socks among others in different colours? Under this condition, the obvious colour cue is no longer available. You need to depend on some other cues. Brightness is not applicable, because no significant brightness difference exists among socks of different colours. Examining the shape sequentially is also impractical, since this property is similar to all socks. Without colour information, it can be difficult to distinguish objects from one another efficiently.

While normal people can regain good colour discrimination with suitable lighting, some are not as fortunate. They are those 8% of the world population which are classified as "colour-blind", or more precisely, "colour deficient". They are well-known for confusing several groups of colours. Consequently, they can come across similar discrimination problems if the objects are of those confusing colours.

Figure 1.1 shows a variety of images randomly picked from a standard image database [16]. Figures 1.2 and 1.3 present the modified versions which simulate the confusions as experienced by the colour deficient people. The views corresponding to colour deficient people are in general less distinctive than those of normal people. For example, the nose of the baboon in the image "baboon" shows similar colour

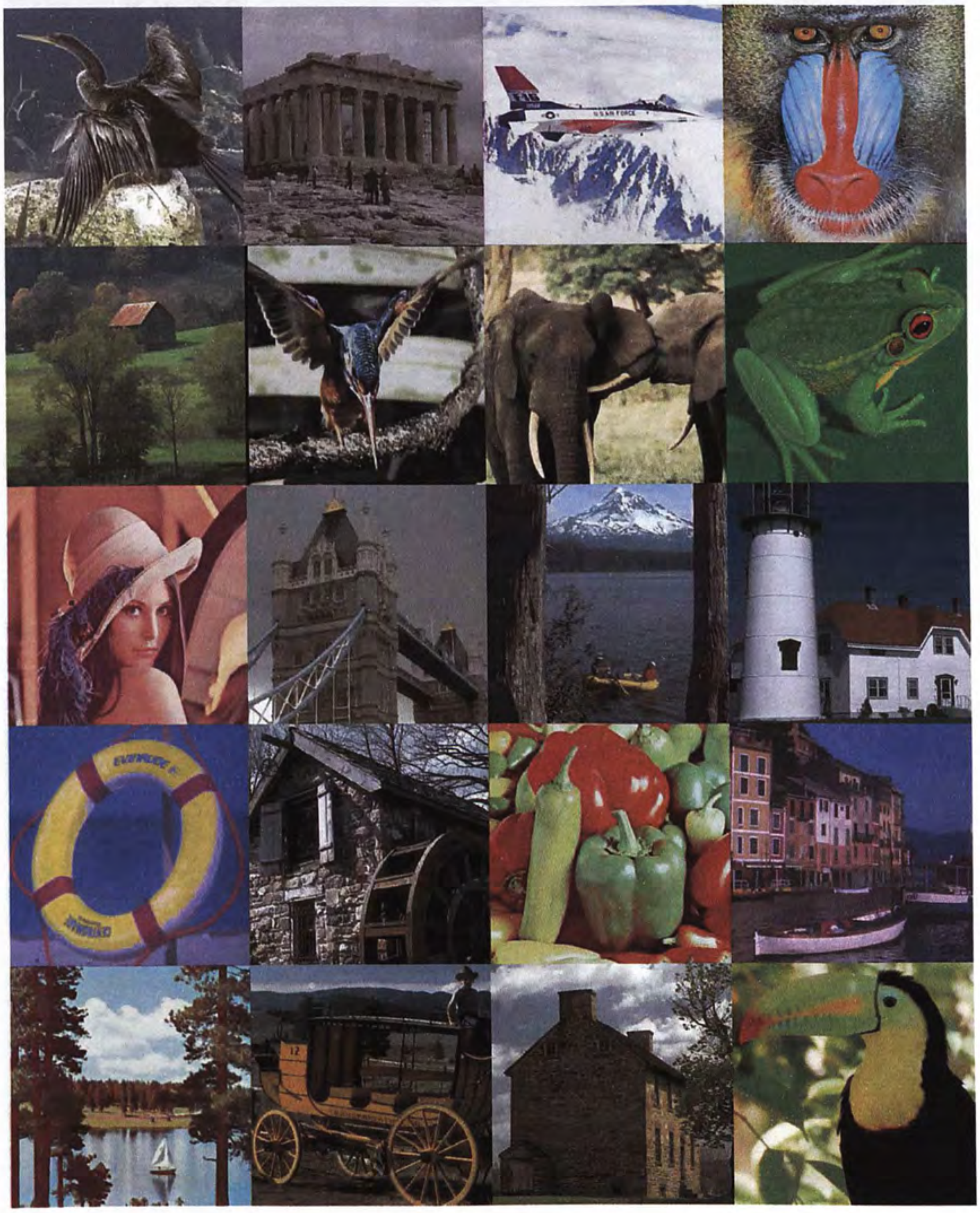


Figure 1.1: A variety of images (selected from [16]) as viewed by normal people. (From left to right) First row: anhinga, athens, avion, baboon. Second row: barnfall, bird, elephant, frog. Third row: lena, london, lostlake, malight. Fourth row: mare, oldmill, peppers, portofino. Fifth row: sailboad, stagcach, stonehse, toucan.

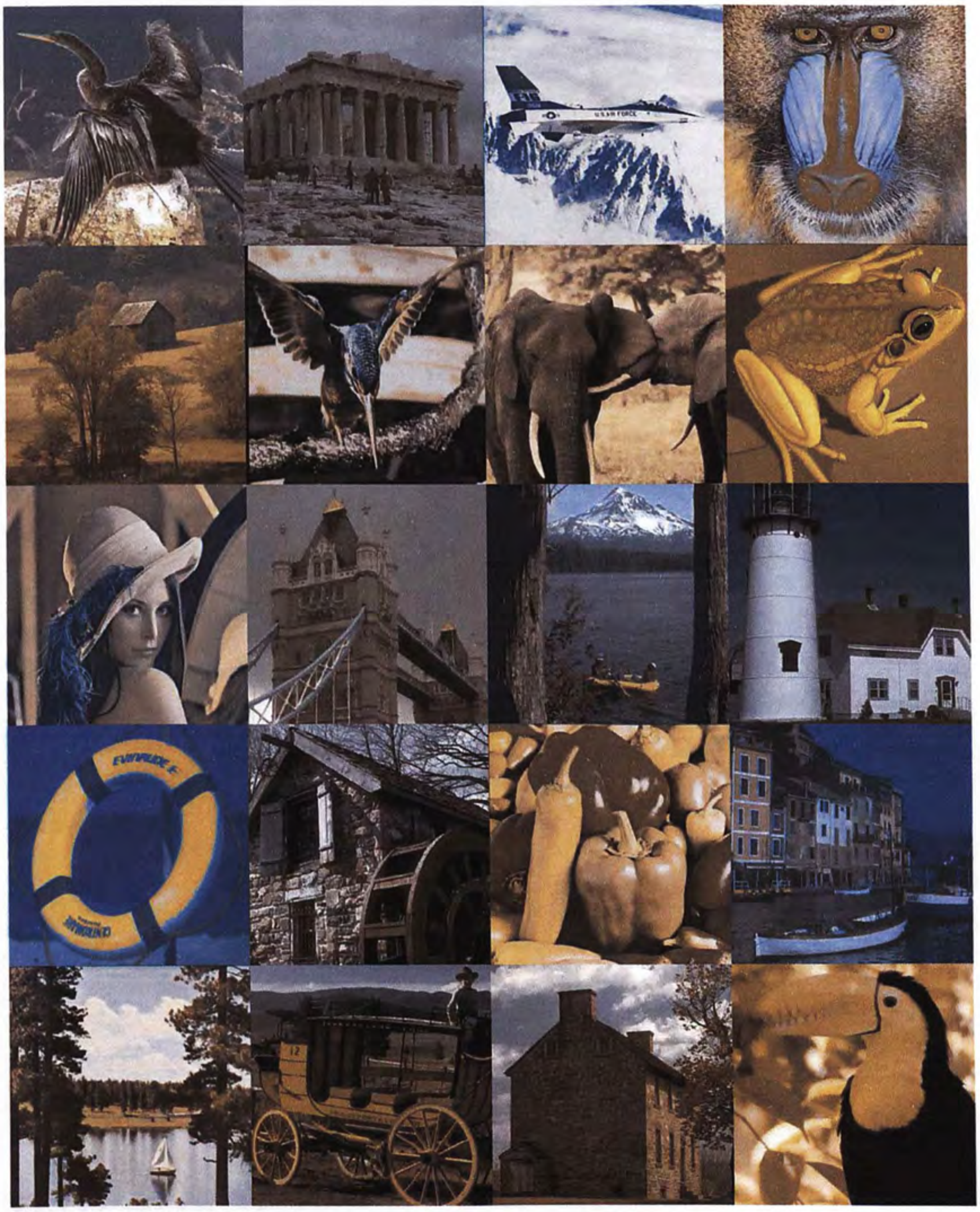


Figure 1.2: Simulated view showing how the variety of images in Figure 1.1 can be confused by people suffering from protanopia, a kind of colour deficiency. (From left to right) First row: anhinga, athens, avion, baboon. Second row: barnfall, bird, elephant, frog. Third row: lena, london, lostlake, malight. Fourth row: mare, oldmill, peppers, portofino. Fifth row: sailboad, stagcach, stonehse, toucan.

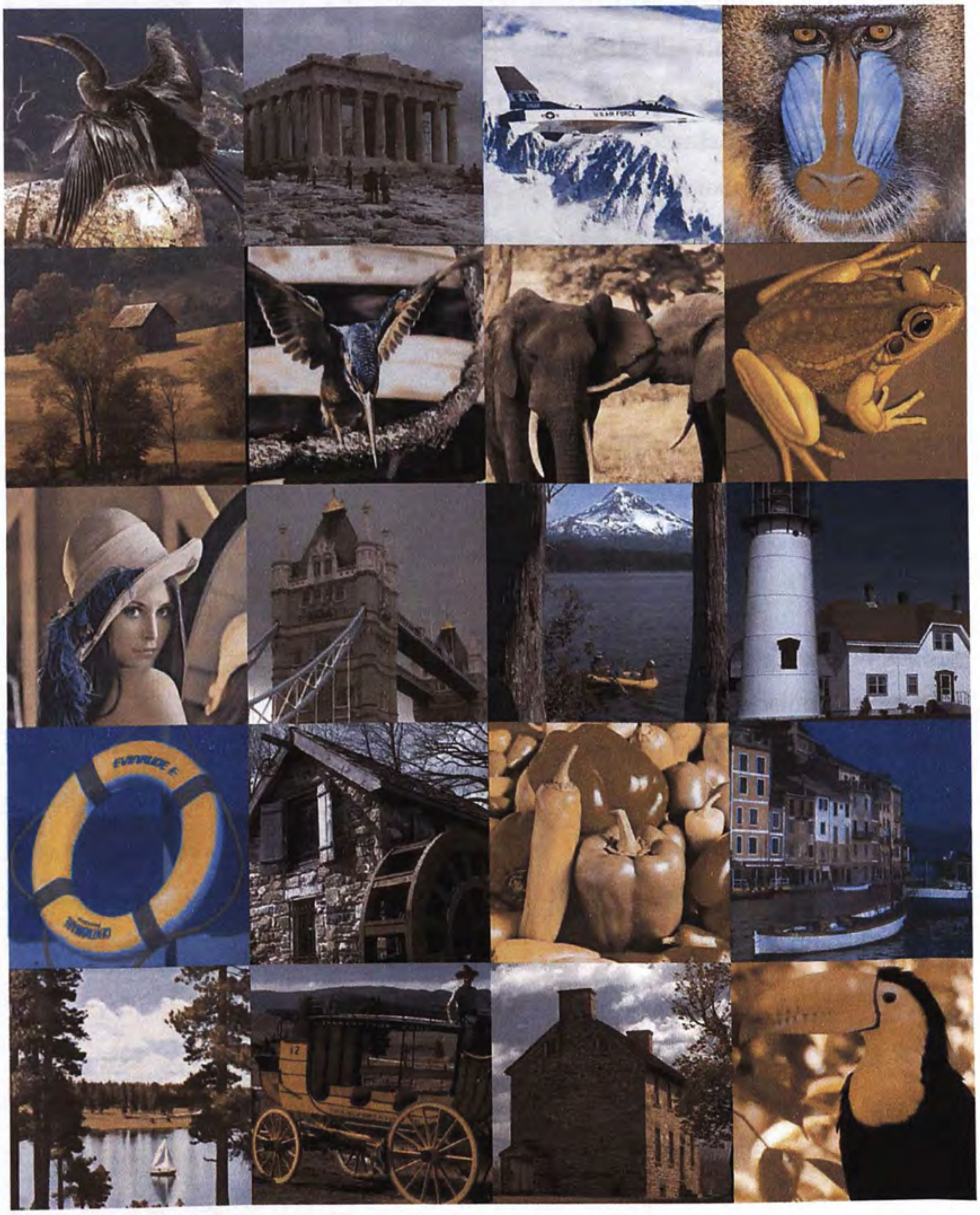


Figure 1.3: Simulated view showing how the variety of images in Figure 1.1 can be confused by people suffering from deuteranopia, another kind of colour deficiency. (From left to right) First row: anhinga, athens, avion, baboon. Second row: barnfall, bird, elephant, frog. Third row: lena, london, lostlake, malight. Fourth row: mare, oldmill, peppers, portofino. Fifth row: sailboad, stagcach, stonehse, toucan.

as the head skin. The eye of the frog in the image "frog" seems to be of the same colour as the body. But what is more problematic is the discrimination loss in some images. Take the image "toucan" as an example. The colourful beak of the toucan now shows monochromatic. The situation of the image "barnfall" is even worse. Not only the roof but also the red leaves now show in similar colour as the other green leaves. While we can still depend on some other cues like the shapes to identify the roof, there is no way to figure out the red leaves from the green leaves. Offering methods to at least redeeming this kind of information loss is thus in need.

With the rapid development of multimedia and telecommunication technologies, the human visual world is now rich of images reproduced with visual display units (VDUs) like cathode ray tubes (CRTs) and liquid crystal display (LCDs). Any handy solution that improves the colour vision experience of the colour-blind over these devices can thus help a lot.

This motivates the development of Stereo Visual Display Unit - Monocular Compensation (SVDU-MC), our colour vision improvement scheme which works with VDUs. The scheme requires the VDUs to be equipped with stereoscopic display capability, which can now be easily achieved with the widely available and affordable stereoscope. Meanwhile, autostereoscopy [5] has also been developed. This technology allows the different images to be fed to the eyes without the use of bulky artificial physical devices like shutter glasses. In other words, the implementation can be very handy.

The stereoscopic display capability is used for displaying simultaneously an unaltered original view to one eye, and a modified compensated view to the other eye. The compensated view provides the discriminations missed in the original view. By combining the discriminations from the two views, the full set of discriminations can be obtained. We suggest that such a presentation strategy, also known as monocular compensation, is worth a study by other researchers aiming to develop aids for the colour-blind population.

Associated with the use of this presentation strategy are some new challenges on the compensation algorithm which generates the compensated view. Among them all, the trade-off between compensation and visual comfort is the most demanding: To offer the missed discriminations, colours in the compensated view must be changed. However, this brings along binocular image dissimilarity and hence visual discomfort. Compensation should thus be offered only when necessary, and according to individual preference. To fulfil these two conflicting criteria, we have designed gamut-based palette compression (GBPC) as the compensation algorithm for use with SVDU-MC. Having the single tuneable parameter c, the algorithm allows compensation to adapt to the needs of different people.

To explain our design, this thesis is organized as follows.

In the next chapter (Chapter 2), we first review the basics of colourimetry which helps in the characterization of colour vision and colour deficiency. This is followed by our arguments for the necessity of monocular compensation in Chapter 3. The same chapter also details the desirable properties of the compensation algorithm working with that presentation strategy. Chapter 4 focuses on the system architecture of SVDU-MC. Emphasis is put on how GBPC has been designed to fulfil the properties discussed in the previous chapter.

A prototype of SVDU-MC has been built and evaluated by five colour deficient people of different kinds and severities. The results showing its improvement capability are presented in Chapter 5, before we give our conclusion and future works in Chapter 6.

Chapter 2

Characterization of Colour Deficiency

To support our subsequent analysis, we present here a review of the mechanism of colour vision and the theory of colourimetry, which are very useful in understanding the patterns observed in discrimination ellipses of colour deficient people. In fact, discrimination ellipse over a perceptually uniform chromaticity diagram is the most widely used tool for the characterizing colour discrimination.

Discrimination ellipse and its derivatives are very useful. Their uses can be seen in the quantification of colour differences, the specification of confusing colour sets, the diagnosis of colour deficiency and even the simulation of colour-blind vision. Those which are related to this thesis are discussed in this chapter.

2.1 Mechanism of colour vision

Modern theory of colour vision describes colour vision as a two-stage process, first the sensory stage and then the perception stage.

The sensory stage starts when an electromagnetic wave with spectral character-

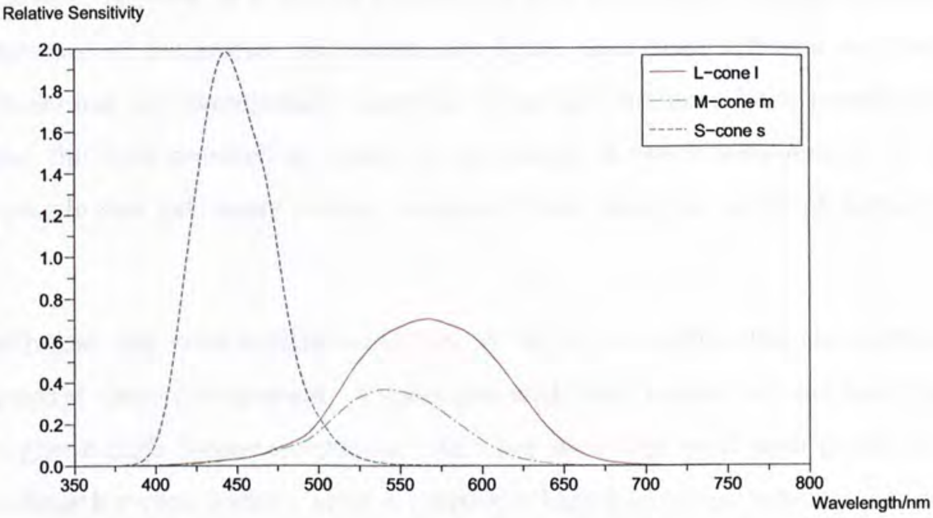


Figure 2.1: Relative sensitivities of the three types of cone cells: l, m and s. (After Stockman et. al. [29])

istics $e(\lambda)$ hits the retina of our eyes. Trichromatic theory, proposed by Maxwell, Young and Helmholtz, suggests that three types of cone cells, classified accordingly to their unique spectral efficiencies, are excited differently.

Figure 2.1 shows the sensitivity curves of these three types of cone cells in a typical human. These cone cells are named according to the spectral range in which they are most sensitive. L-cone, with sensitivity $l(\lambda)$, is most sensitive to long wavelength (red region) of the visible spectrum. M-cone, with sensitivity $m(\lambda)$, is most sensitive to middle wavelength (green region). S-cone, with sensitivity $s(\lambda)$, is most sensitive to short wavelength (blue region). Equations 2.1 to 2.3 show how the excitation levels of the L-cone (L), M-cone (M) and S-cone (S) are related to their respective sensitivities and the electromagnetic wave exciting them.

$$S = \int_{\lambda} e(\lambda)s(\lambda)d\lambda \tag{2.1}$$

$$M = \int_{\lambda} e(\lambda)m(\lambda)d\lambda \tag{2.2}$$

$$L = \int_{\lambda} e(\lambda)l(\lambda)d\lambda \tag{2.3}$$

As a result, different spectra are just different sets of $[S \;\; M \;\; L]$ values after the sensory stage. A piece of evidence supporting this tristimulus coding mechanism is the presence of metamers. Metamers are lights that have different wavelength distributions but are perceptually identical. It would not have been possible if we had coded the light received in terms of its powers at every wavelength. In fact, normal people can get many colours matched with additive mixes of some three colours.

Nevertheless, the cone excitation triplet $[S\ M\ L]$ can reflect the characteristics of the spectral content in general. A spectrum with high power in short wavelength tends to give a high S-cone excitation. Another spectrum with high power in the long wavelength region instead gives a relatively high L-cone excitation.

However, we do not describe our colour experience with those three quantities corresponding to the intensities of long, middle and short wavelengths. Luminance and chromaticity are much more commonly used. Luminance describes the overall light intensity, while chromaticity is for the description of colour independent of luminance. Chromaticity can be further divided into two attributes: hue and saturation. Hue describes how a visual sensation of a colour is close to red, green or blue, or a combination of any two of them. It is found to be determined by the dominant wavelength at which the spectral power is a maximum. Saturation tells how the visual sensation of the hue is mixed with the "white" light. The lower the saturation, the more muted and grey the hue appears to the viewer.

We observe the following phenomena when we describe colours in these terms: First, we can describe chromaticity components (hue and saturation) confidently without being affected by luminance. Second, we claim a luminance increase with a rise in power at any hue and hence wavelength. Third, it is the non-coexistences of some colours known as opposing colours. We never claim any red in green, or blue in yellow, or vice versa.

All these imply a processing model in which the sum and difference of the excitation levels are evaluated. Figure 2.2 shows the details of such a processing network

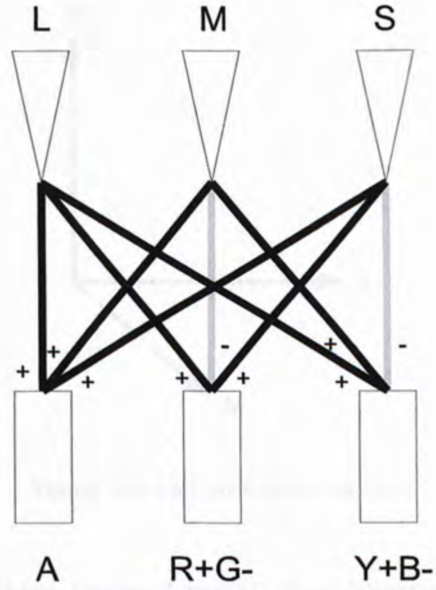


Figure 2.2: Encoding of cone excitations into opponent colour signals in the human visual system. (After Fairchild [7])

as proposed in Hering's opponent process theory. The theory suggests that the perceptual outputs of colour vision include a single achromatic component A and two chromatic components $R+G-$ and $Y+B-$. The contributions of all cone types to the perceptual block A are all additive, thus matching our observed relationship between luminance and spectral power. The comparative nature of the chromatic blocks $R+G-$ and $Y+B-$ explains the non-coexistences of the opponent colour pairs.

2.2 Quantitative specification of colour

The origin of any colour sensation is the spectral content of light. So the most direct way to specify a colour is to record the light intensity at every frequency or wavelength. Though preserving the primary information, this method imposes a high demand on storage space.

A justified alternative is to use a triplet whose components correspond to the

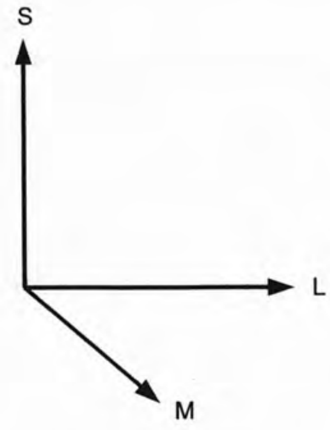


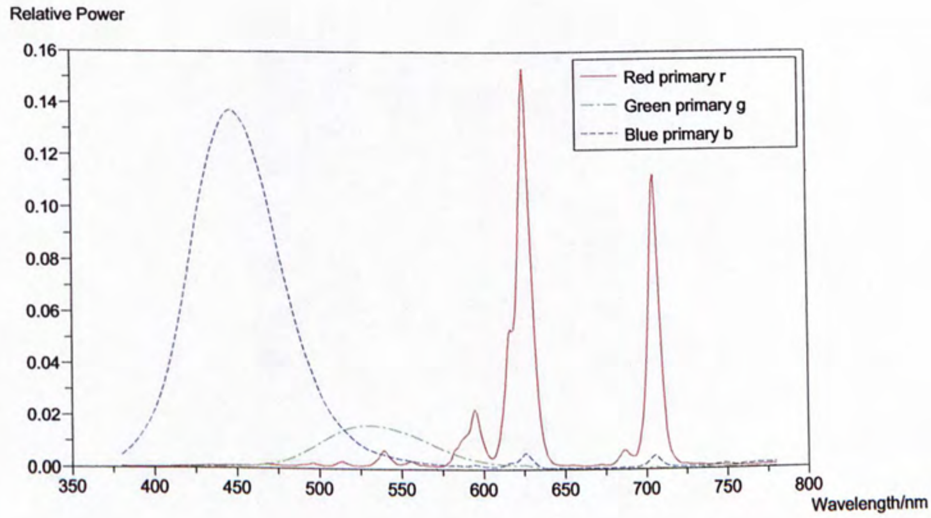
Figure 2.3: SML cone excitation space.

excitation levels of the three types of cones. It is because after the sensory stage, all lights reduce to three excitation values anyway.

Geometrically, colours coded in terms of cone excitation triplets can be represented in the SML cone excitation space. The space is spanned by three orthogonal bases, each of which refers to the excitation level of one kind of cone cells (Figure 2.3). In this 3D colour space, a colour is represented as a vector, which starts from the origin and ends at the coordinates corresponding to the excitation values produced by that colour.

Transformations from this colour space to some others are available. Examples of these alternative colour spaces include RGB and CIE XYZ, with their respective colour matching functions shown in Figure 2.4. Each of these colour matching functions corresponds to a colour primary. In these colour spaces, we use a triplet known as tristimulus value to specify a colour. By mixing the colour primaries according to the amounts specified in the triplet, we can reproduce a spectrum which offers the required colour sensation.

The transformations between these colour spaces are linear. Each of these trans-



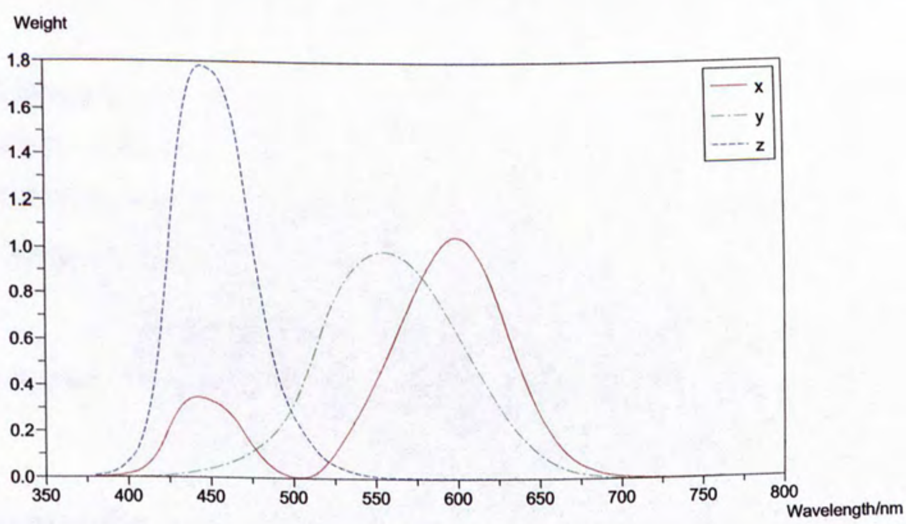


Figure 2.4: RGB and CIE XYZ 1931 (2° field) colour matching functions. Top: RGB colour matching functions for a typical CRT: r, g and b. They correspond to the emission spectrum of the CRT red primary, green primary and blue primary respectively. (After Golz [8]). Bottom: CIE XYZ 1931 (2° field) colour matching functions: x, y and z.

formations, as shown in Equations 2.4 to 2.7, features a matrix operation.

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = T_{RGB \to SML} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \tag{2.4}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = T_{SML \to RGB} \begin{bmatrix} S \\ M \\ L \end{bmatrix} \tag{2.5}$$

$$\begin{bmatrix} S \\ M \\ L \end{bmatrix} = T_{XYZ \to SML} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{2.6}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = T_{SML \to XYZ} \begin{bmatrix} S \\ M \\ L \end{bmatrix} \tag{2.7}$$

The relationship among the transformation matrix $T_{XYZ \to SML}$, the respective cone sensitivity functions, and CIE XYZ colour matching functions is shown in Equation 2.8. Each matrix coefficient corresponds to the contribution of a particular CIE XYZ component to the cone excitation in concern.

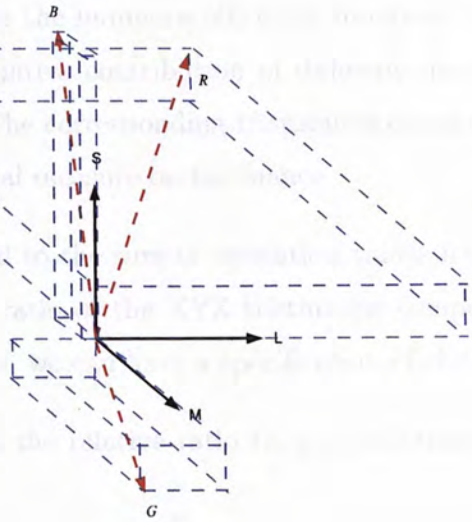
$$T_{XYZ \to SML} = \begin{bmatrix} \int_\lambda x(\lambda)s(\lambda)d\lambda & \int_\lambda y(\lambda)s(\lambda)d\lambda & \int_\lambda z(\lambda)s(\lambda)d\lambda \\ \int_\lambda x(\lambda)m(\lambda)d\lambda & \int_\lambda y(\lambda)m(\lambda)d\lambda & \int_\lambda z(\lambda)m(\lambda)d\lambda \\ \int_\lambda x(\lambda)l(\lambda)d\lambda & \int_\lambda y(\lambda)l(\lambda)d\lambda & \int_\lambda z(\lambda)l(\lambda)d\lambda \end{bmatrix} \tag{2.8}$$

The corresponding reverse transformation is given by the respective inverse shown in Equation 2.9.

$$T_{SML \to XYZ} = (T_{XYZ \to SML})^{-1} \tag{2.9}$$

The transformation matrices converting between RGB and SML colour spaces $T_{RGB \to SML}$ and $T_{SML \to RGB}$ are defined similarly. They are shown in Equations 2.10 and 2.11.

$$T_{RGB \to SML} = \begin{bmatrix} \int_\lambda r(\lambda)s(\lambda)d\lambda & \int_\lambda g(\lambda)s(\lambda)d\lambda & \int_\lambda b(\lambda)s(\lambda)d\lambda \\ \int_\lambda r(\lambda)m(\lambda)d\lambda & \int_\lambda g(\lambda)m(\lambda)d\lambda & \int_\lambda b(\lambda)m(\lambda)d\lambda \\ \int_\lambda r(\lambda)l(\lambda)d\lambda & \int_\lambda g(\lambda)l(\lambda)d\lambda & \int_\lambda b(\lambda)l(\lambda)d\lambda \end{bmatrix} \tag{2.10}$$



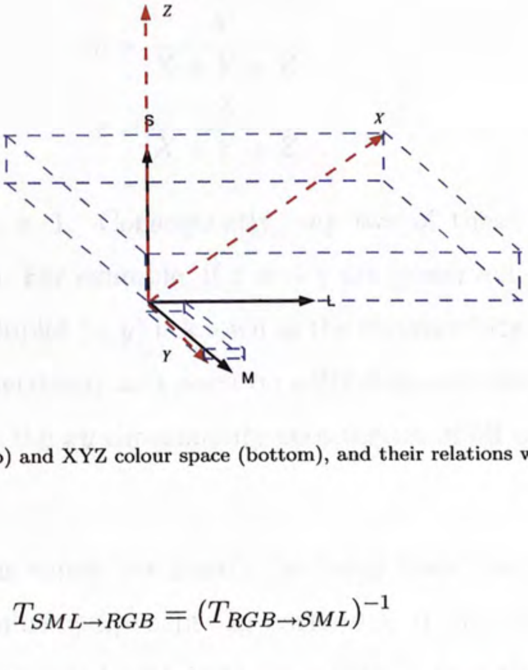


Figure 2.5: RGB colour space (top) and XYZ colour space (bottom), and their relations with SML colour space.

$$T_{SML \to RGB} = (T_{RGB \to SML})^{-1} \tag{2.11}$$

Geometrically, the transformations are equivalent to changing the directions of the bases about the same origin (Figure 2.5).

RGB colour space is primarily designed for VDUs: A RGB digital code triplet represents the intensities of the three colour primaries which produce of a specific colour in these electronic devices. In contrast, other colour spaces have some perceptual meaning. In particular, the XYZ colour spaces have one colour matching function, $y(\lambda)$, which resembles the luminous efficiency function. The luminous efficiency function shows the relative contribution of different spectral components on the perceived brightness. The corresponding tristimulus component, the CIE Y value, thus provides a numerical measure on luminance.

Luminance is closely related to the sum of excitation values from different cone types. So by calculating the ratio of the XYZ tristimulus components (in which absolute values are eliminated), we can have a specification of chromaticity.

For XYZ tristimulus value, the relative ratio (x, y, z) is found using Equations 2.12 to 2.14.

$$x = \frac{X}{X+Y+Z} \tag{2.12}$$

$$y = \frac{Y}{X+Y+Z} \tag{2.13}$$

$$z = \frac{Z}{X+Y+Z} \tag{2.14}$$

This implies $x + y + z = 1$. Consequently, any two of these three values are sufficient to code the ratio. For example, if x and y are preserved, z can be readily found with $1-x-y$. The duplet (x, y) is known as the chromaticity coordinates, and it can be represented geometrically as a point on a 2D diagram called a chromaticity diagram. Figure 2.6 shows the xy chromaticity coordinates of all colours in the real world.

The absolute tristimulus values can readily be found from that duplet accompanied with any one tristimulus component. In particular, if the selected tristimulus component is positively correlated with luminance (like Y in XYZ space), we have a triplet which represents chromaticity and luminance separately. Equations 2.15 to 2.17 shows how the XYZ tristimulus value can be recovered from such a triplet (x, y, Y).

$$X + Y + Z = \frac{Y}{y} \tag{2.15}$$

$$X = x(X + Y + Z) = x\frac{Y}{y} \tag{2.16}$$

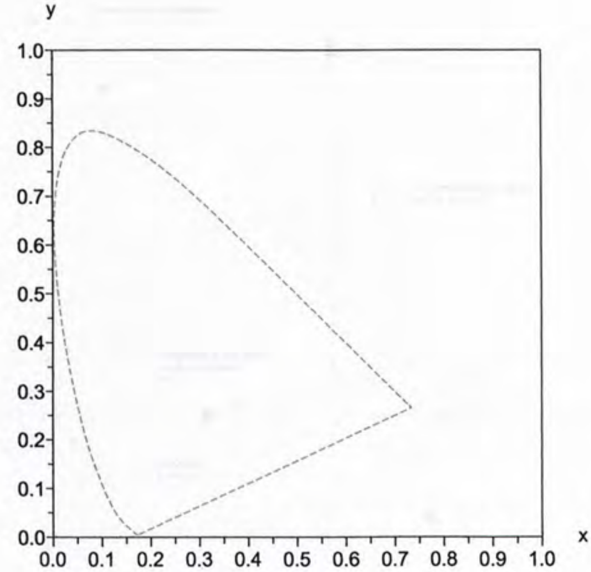


Figure 2.6: xy **chromaticity coordinates of all colours in the real world (Enclosed in the convex tongue shape).**

$$Z = z(X + Y + Z) = z\frac{Y}{y} \tag{2.17}$$

Geometrically, the (x, y) chromaticity coordinates of a colour can be found from the intersection of the colour vector and the unit plane $X + Y + Z = 1$ in the 3D XYZ colour space. The xy chromaticity diagram, which is a 2D plot of the (x, y) chromaticity coordinates, is the orthographic projection of this unit plane onto the XY plane [19, 21]. Figure 2.7 summarizes the relationship among these concepts.

An interesting property of the chromaticity diagram is that if a chromaticity (x_0, y_0) lies on the line bounded by chromaticities (x_1, y_1) and (x_2, y_2), the colour represented by (x_0, y_0) is readily reproduced through an additive mix of the two colours corresponding to (x_1, y_1) and (x_2, y_2) in some ratio. This phenomenon can be explained in terms of colour vectors: The colour vectors corresponding to the series of colours spanned by (x_1, y_1) and (x_2, y_2) can each be represented by a particular linear combination of the colour vector bases corresponding to (x_1, y_1) and (x_2, y_2).

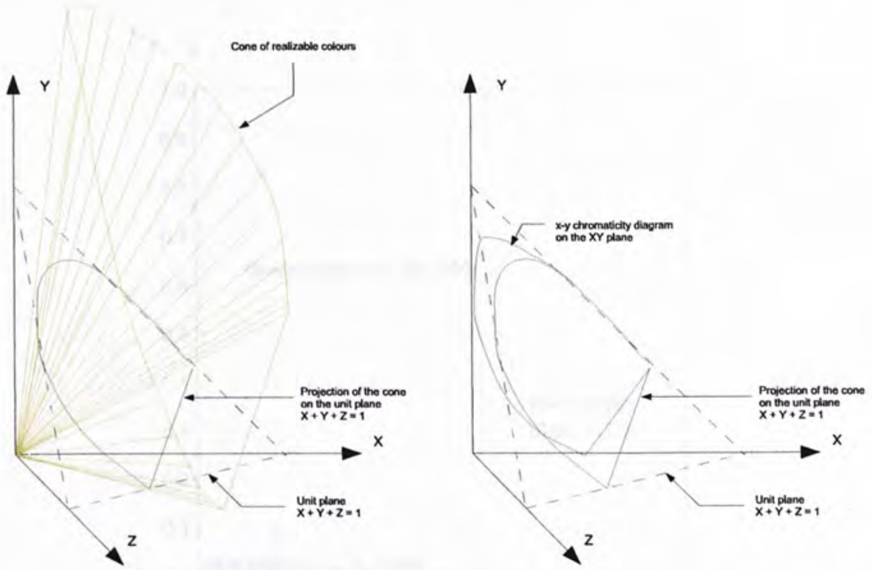


Figure 2.7: Colour vectors in the cone excitation space and chromaticity coordinates in the xy chromaticity diagram.

A VDU utilizes additive mixes of its three colour primaries to reproduce a variety of colours. As a result, the chromaticities that it can reproduce are bounded by a triangular display gamut with vertices being the chromaticity coordinates of the three colour primaries (Figure 2.8).

2.3 Discrimination ellipses

Colour deficient people confuse specific sets of colours when they are of no significant luminance difference. This suggests a deviation in their chromatic sensitivity from the normal. This thus validates the use of the chromaticity diagram to characterize colour discrimination, or more precisely, chromatic discrimination.

A discrimination ellipse is used for indicating the set of confusing chromaticities. Centred at a reference chromaticity (x_c, y_c), the ellipse encloses those neighbouring chromaticities which are indistinguishable from (x_c, y_c) when no luminance difference exists. The discrimination ellipses of normal people over the xy chromaticity

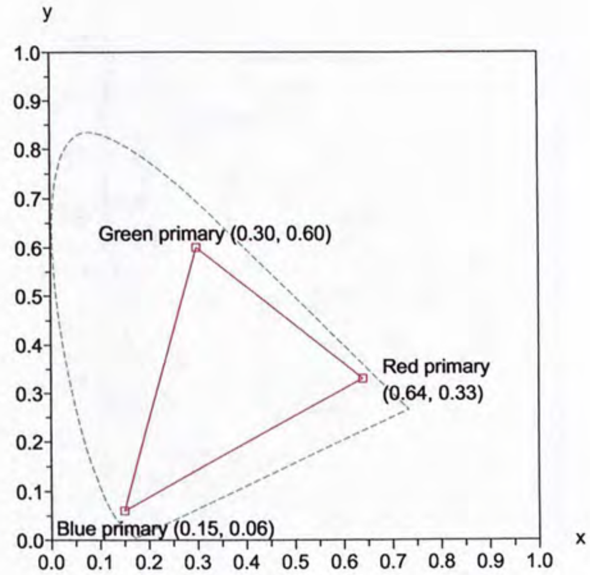


Figure 2.8: Display gamut of a standard CRT compared with all possible chromaticities. The CRT gamut is defined by the solid triangle, while the gamut realizable by natural lights is bounded by the dash tongue shape.

diagram have been plotted [18]. The diagram is reproduced in Figure 2.9.

Discrimination ellipses are closely related to the cone excitation space. Given a colour vector, one can find the associated boundary colour vectors which refer to the boundary colours that are just differentiable from the reference colour in different directions. When these boundary colour vectors are projected onto the unit plane $S + M + L = 1$, discrimination ellipses are formed. (Figure 2.10)

The ellipses in the xy chromaticity diagram are not all circular and of the same size over the gamut. Quantifying chromaticity difference then needs the specification of the gamut region in addition to the Euclidean distance between the two chromaticities. This is inconvenient.

To improve the situation, a linear transform from xy coordinate system to CIE $u'v'$ (1976) coordinate system has been purposed. Details of the transformations are

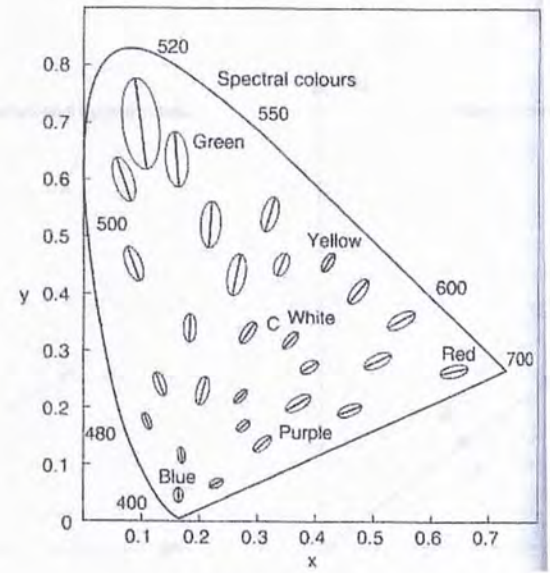


Figure 2.9: Discrimination ellipses of a normal person in the xy chromaticity diagram. The ellipses are enlarged approximately 10 times. (After MacAdam [18])

shown in Equations 2.18 to 2.21.

$$u' = \frac{4x}{-2x + 12y + 3} \tag{2.18}$$

$$v' = \frac{9y}{-2x + 12y + 3} \tag{2.19}$$

$$x = \frac{9u'}{6u' - 16v' + 12} \tag{2.20}$$

$$y = \frac{4v'}{6u' - 16v' + 12} \tag{2.21}$$

Figure 2.11 and 2.12 show the axes of some discrimination ellipses in xy and $u'v'$ coordinate system respectively. In the $u'v'$ coordinate system, the ellipses are of approximately equal size, with radius of 1 JND (just noticeable difference) in all directions. So in this coordinate system, the Euclidean distance between some two chromaticity coordinates is sufficient to specify the chromaticity difference.

A derivative of the $u'v'$ chromaticity coordinate system is the CIE $L^*u^*v^*$ colour space. With Y_n, u'_n and v'_n being the CIE Y value and (u', v') coordinates of the maximum-luminance white (also known as reference white) in the environment, the

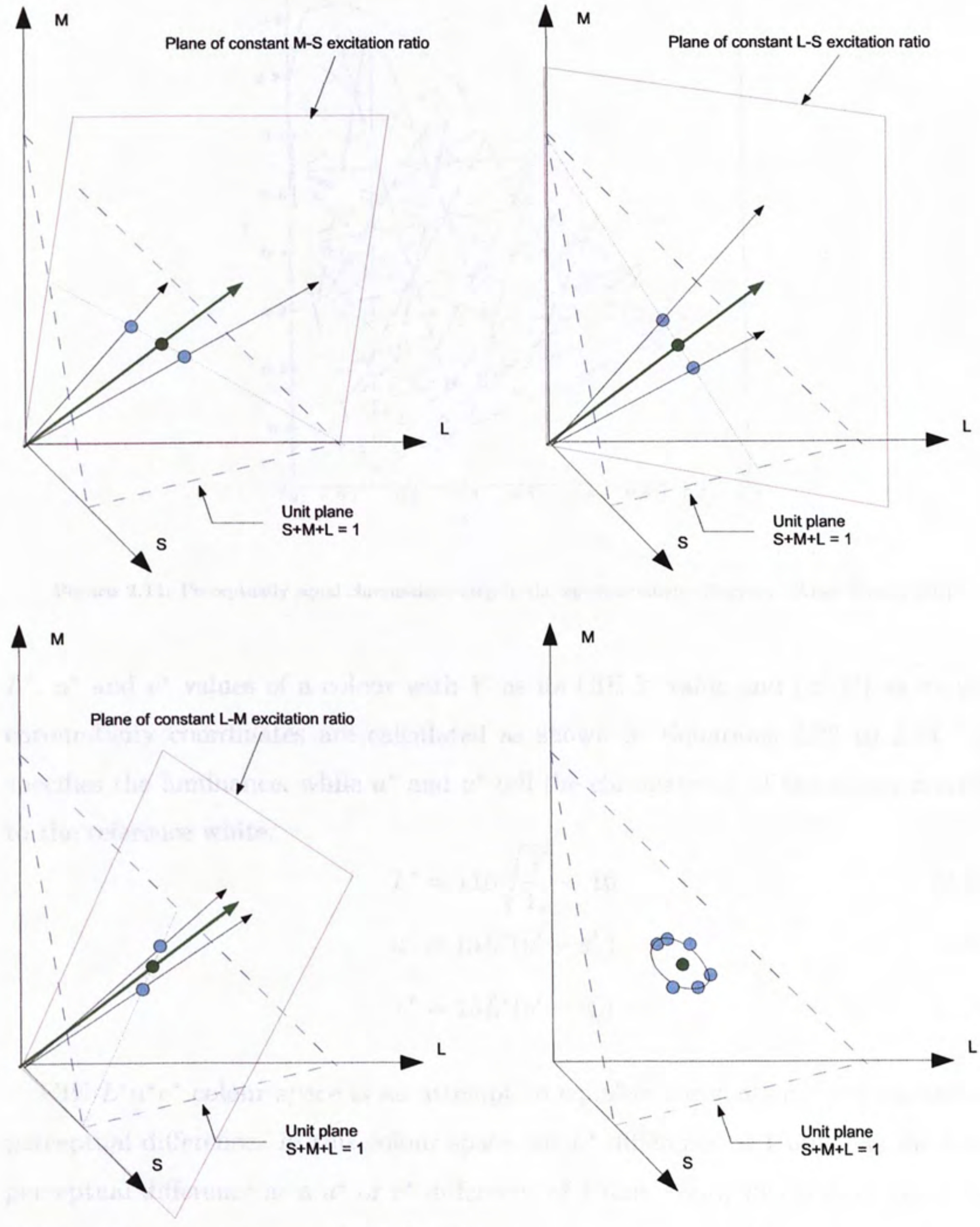


Figure 2.10: Boundary colour vectors in the cone excitation space and discrimination ellipses in the xy chromaticity diagram. The green vectors and dots refer to the reference colours and chromaticities respectively.

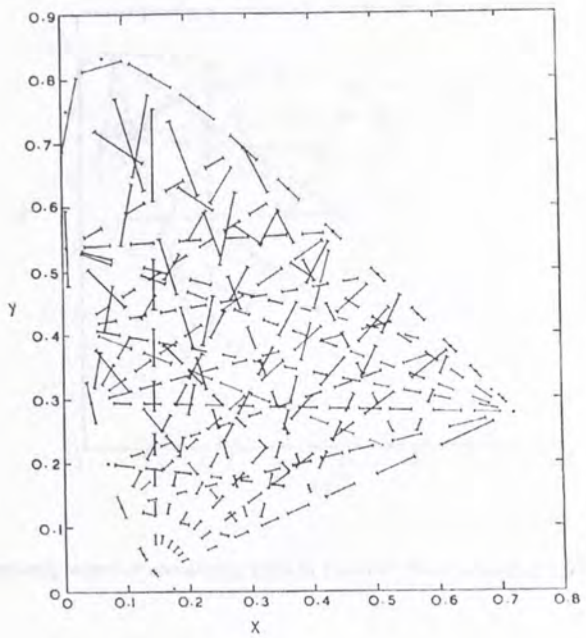


Figure 2.11: Perceptually equal chromaticity step in the xy chromaticity diagram. (After Wright [31])

L^*, u^* and v^* values of a colour with Y as its CIE Y value and (u', v') as its $u'v'$ chromaticity coordinates are calculated as shown in Equations 2.22 to 2.24. L^* specifies the luminance, while u^* and v^* tell the chromaticity of the colour relative to the reference white.

$$L^* = 116\sqrt[3]{\frac{Y}{Y_n}} - 16 \tag{2.22}$$

$$u^* = 13L^*(u' - u'_n) \tag{2.23}$$

$$v^* = 13L^*(v' - v'_n) \tag{2.24}$$

CIE $L^*u^*v^*$ colour space is an attempt to equalize chromaticity and luminance perceptual difference. In this colour space, an L^* difference of 1 unit has the same perceptual difference as a u^* or v^* difference of 1 unit. With this goal in mind, the chromaticity components u^*, v^* in this space are in proportion to the luminance L^*. This is consistent with the fact that colourfulness increases with the luminance level. The Euclidean distance of any two CIE $L^*u^*v^*$ value, ΔE^*_{uv}, gives the overall perceptual difference between the two colours.

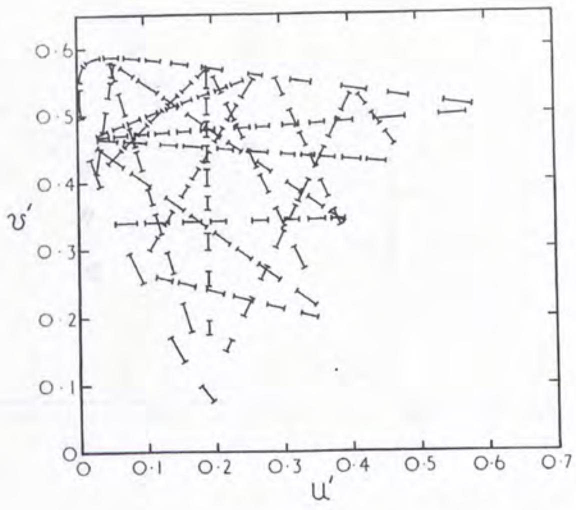


Figure 2.12: Perceptually equal chromaticity step in the $u'v'$ chromaticity diagram. (After Hunt [15])

$$\Delta E^*_{uv} = \sqrt[2]{(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2} \qquad (2.25)$$

2.4 Colour perception of colour deficient people

Discrimination ellipses of a number of colour deficient people have been found from experiments [26], and they are reproduced in Figures 2.13 to 2.16. As expected, the discrimination ellipses of the colour-blind suffer from elongations as a result of their degraded chromatic discrimination.

The discrimination power along different chromatic directions can be affected differently by these elongations. Figure 2.17 shows qualitatively how the minimum detectable chromaticity difference is affected by θ which is the angle tilted from the elongated ellipse axis by the chromaticity pair. The larger the deviation, the less the chromaticity difference needed. So, the angle θ can be used as a measure of how the chromatic discrimination power along that particular direction is degraded by colour deficiency.

Figure 2.13: Discrimination ellipses obtained for five subjects with normal colour vision. (After Regan et. al. [26])

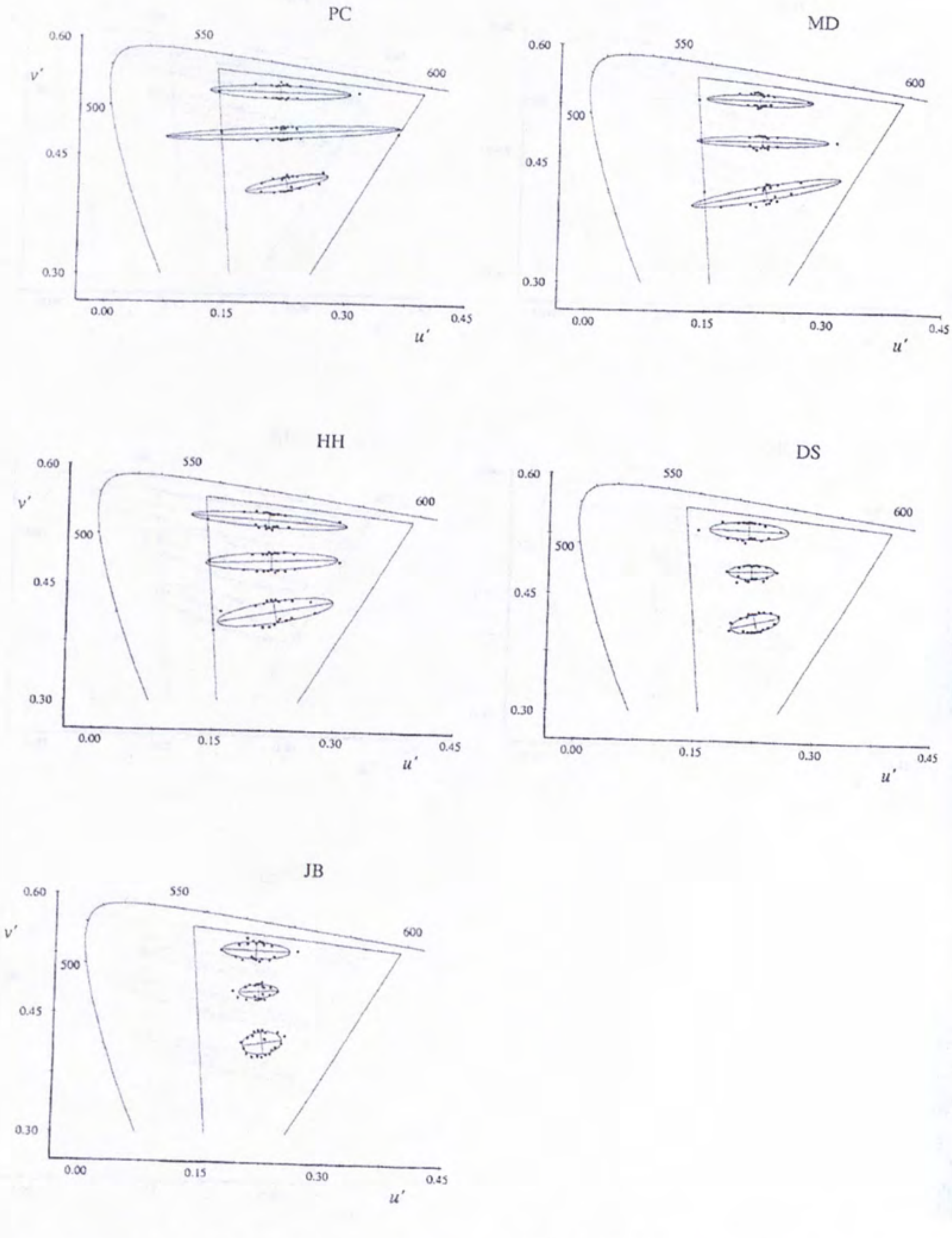


Figure 2.14: Discrimination ellipses obtained for one protanopic subject (PC), one severe protanomalous subject (MD) and three mild protanomalous subjects (HH, DS and JB). (After Regan et. al. [26])

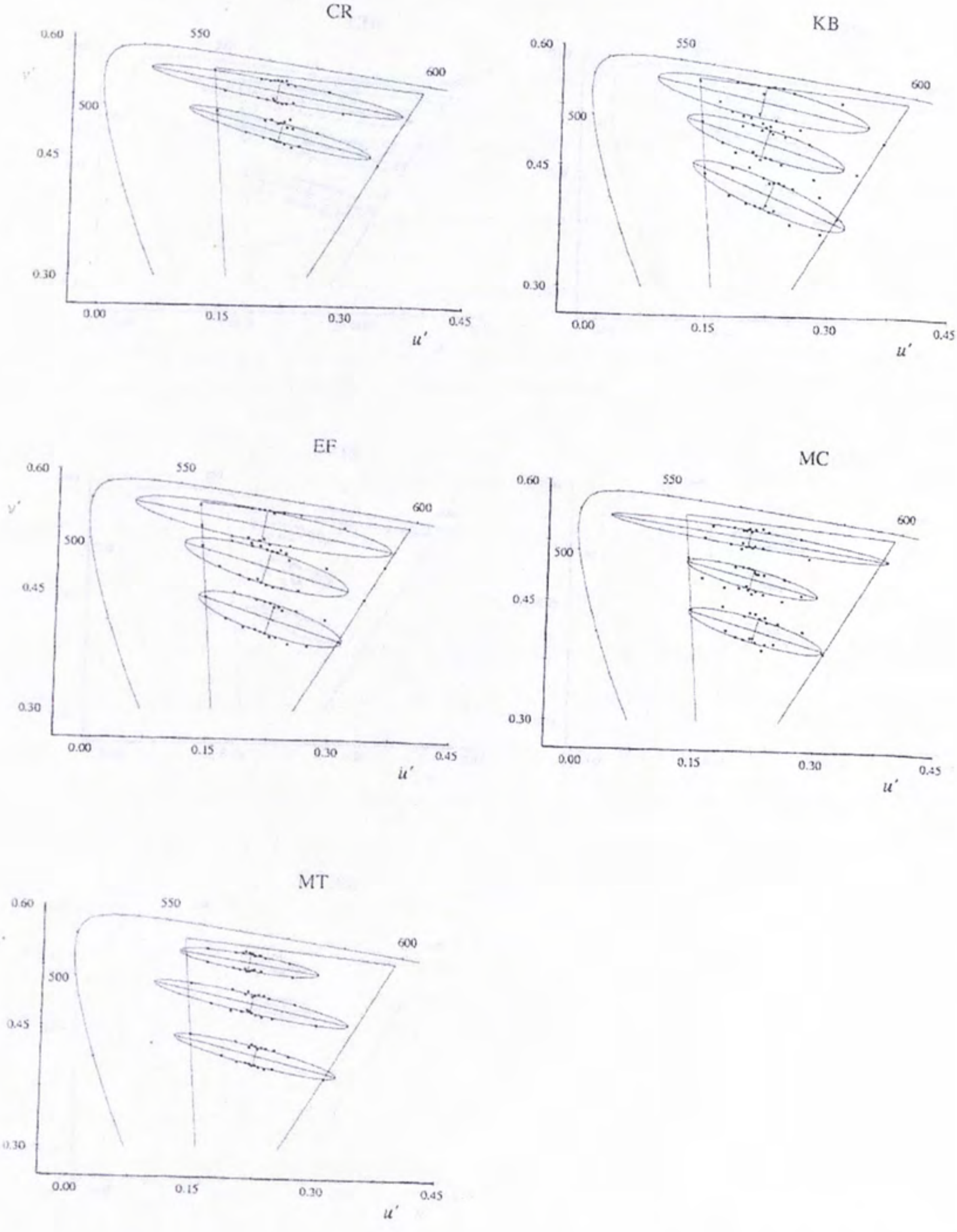


Figure 2.15: Discrimination ellipses obtained for two deuteranopic subjects (CR and KB) and three severe deuteranomalous subjects (EF, MC and MT). (After Regan et. al. [26])

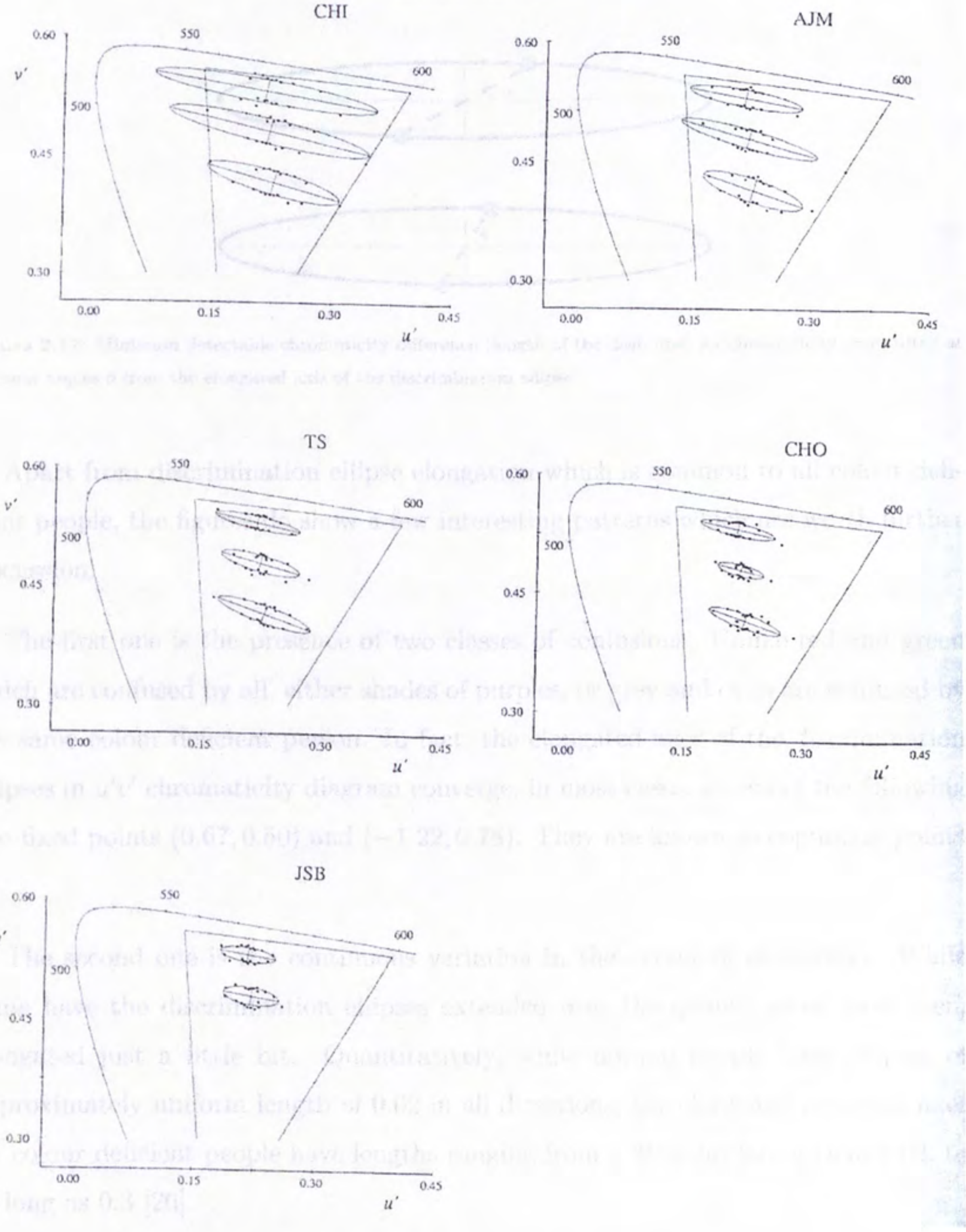


Figure 2.16: Discrimination ellipses obtained for five mild deuteranomalous subjects. (After Regan et. al. [26])

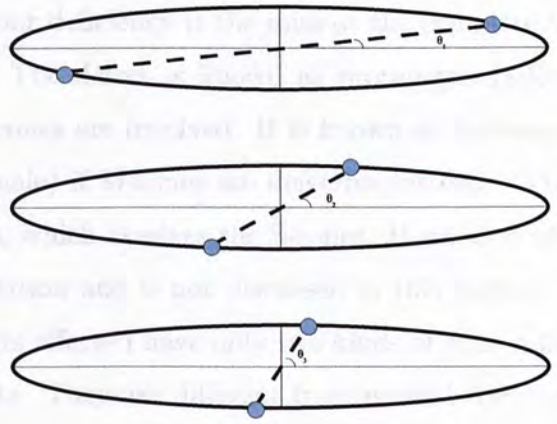


Figure 2.17: Minimum detectable chromaticity difference (length of the dash line) for chromaticity pairs tilted at different angles θ from the elongated axis of the discrimination ellipse.

Apart from discrimination ellipse elongation which is common to all colour deficient people, the figures do show a few interesting patterns which are worth further discussion.

The first one is the presence of two classes of confusions. Unlike red and green which are confused by all, either shades of purples, or grey and cyan are confused by the same colour deficient person. In fact, the elongated axes of the discrimination ellipses in $u'v'$ chromaticity diagram converge, in most cases, to one of the following two fixed points $(0.67, 0.50)$ and $(-1.22, 0.78)$. They are known as copunctal points [2].

The second one is the continuous variation in the extent of elongation. While some have the discrimination ellipses extended over the gamut, some have them elongated just a little bit. Quantitatively, while normal people have ellipses of approximately uniform length of 0.02 in all directions, the elongated principal axes for colour deficient people have lengths ranging from a little bit larger than 0.02, to as long as 0.3 [26].

These observations can be explained with our current understanding on the biological origin of colour deficiency.

One cause of colour deficiency is the miss or the complete failure of a particular type of cone cells. The defect is known as protanopia (affecting 1.0% male and 0.01% female) if L-cones are involved. It is known as deuteranopia (affecting 1.0% male and 0.01% female) if M-cones are defective instead. (There is the third kind known as tritanopia, which involves the S-cones. However it affects an insignificant proportion of population and is not discussed in this thesis.) The protanopic and deuteranopic patients affected have only two kinds of cone cells and are collectively known as dichromats. They are different from normal trichromats who have three kinds of cone cells.

As a result of this miss or complete failure, colours which produce different excitations to the missed cone type but not the other cone types now look the same to the colour deficient people. Drawn on the 3D cone excitation space, these colours form an infinite number of vectors lying on the same plane, known as the plane of constant M-S excitation ratio for the L-cone defect, or the plane of constant L-S excitation ratio for the M-cone defect (Figure 2.18). The consequence, if following the derivation in Figure 2.7, is an infinitely elongated discrimination ellipse in the chromaticity diagram [19, 21].

The directions of these infinite elongations are often indicated by confusion lines. All chromaticities along the same confusion line are indistinguishable from one another to the colour deficient people, if no noticeable luminance difference exists among them.

The convergence of discrimination ellipse elongation or confusion lines can be explained after drawing a few planes of constant M-S excitation ratio for the protanopia case, or planes of constant L-S excitation ratio for the deuteranopia case. As shown in Figure 2.19, the intersect lines of these planes with the $S + M + L = 1$ unit plane converge to a vertex of the triangular unit plane. The vertex is the intersection of the unit plane and the L-cone basis for the protanopic case, or the intersection of the unit plane and the M-cone basis for the deuteranopic case. The convergence holds even after linear transformation of the colour space from SML to XYZ, projection

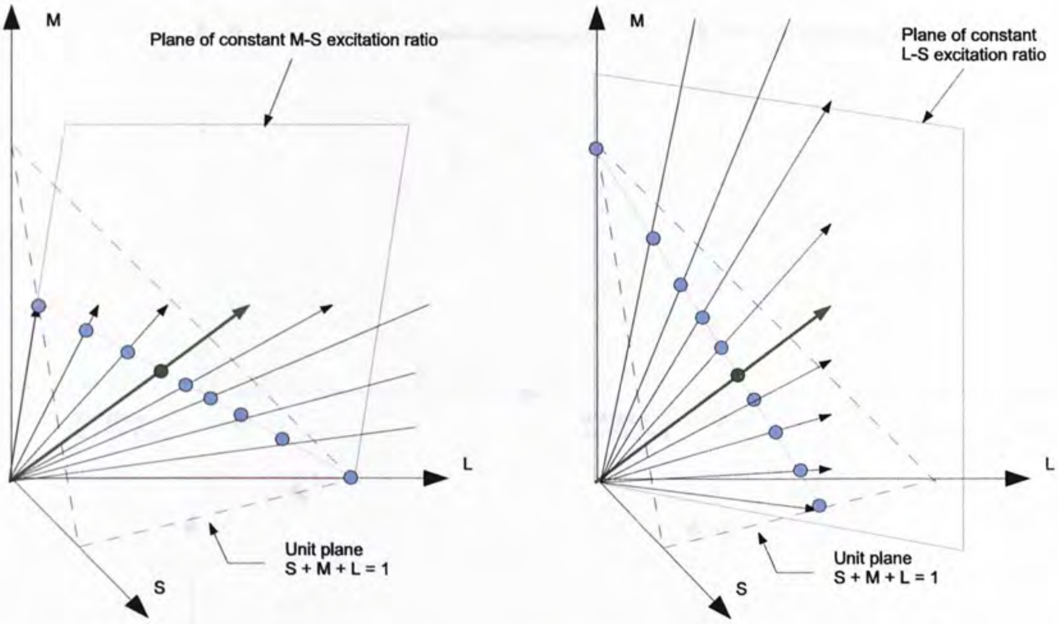


Figure 2.18: An infinite number of colours being perceptually the same to people suffering from protanopia (left) and deuteranopia (right). The green vectors and dots refer to the reference colours and chromaticities respectively.

of the unit plane on the XY plane, and linear transformation to the $u'v'$ perceptual chromaticity coordinate system [21].

Another known cause of colour deficiency is a shift in the sensitivity function of a particular cone cell type. The defect is known as protanomaly (affecting 1.0% male and 0.03% female) if L-cone sensitivity is shifted towards that of M-cone. It is known as deuteranomaly (affecting 5.0% male and 0.35% female) if the defect involves M-cone sensitivity shifted towards that of L-cone instead. These protanomalous and deuteranomalous patients are collectively named anomalous trichromats to indicate the abnormal behaviours of their cone cells.

The consequence of either shift is an increase in the correlation between L-cone and M-cone excitations. Comparing L and M cone excitations has been stated in Hering's opponent process theory as a major perception mechanism. This implies that a larger marginal excitation on the shifted cone type is needed to signal the same perceptual difference.

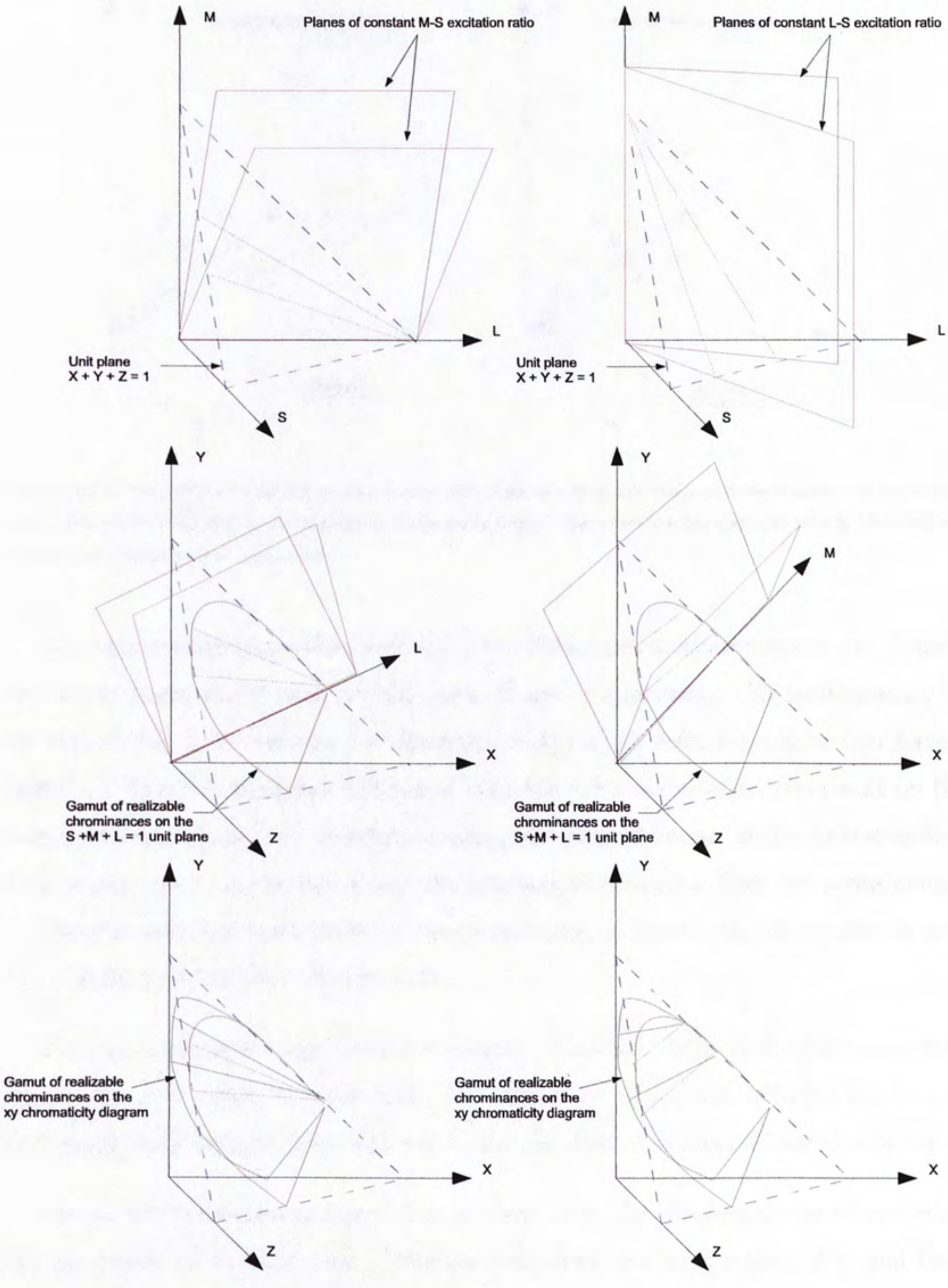


Figure 2.19: Convergence of confusion lines derived from the cone excitation space. Protanopia (left). Deuteranopia (right).

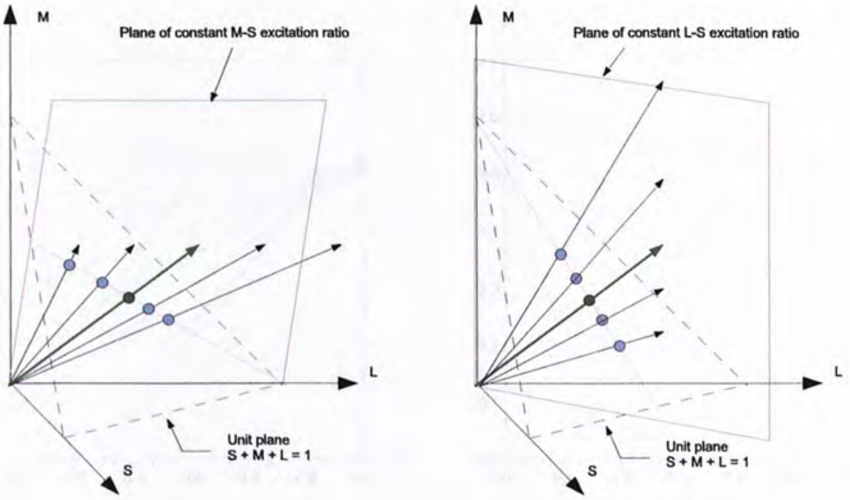
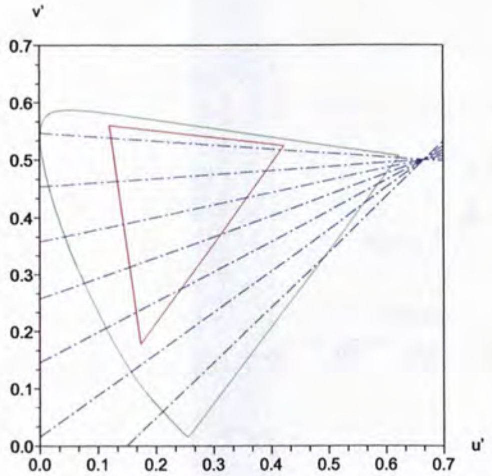


Figure 2.20: Just differentiable colours and chromaticities about a reference colour and chromaticity for protanomalous trichromats (left) and deuteranomalous trichromats (right). The green vectors and dots refer to the reference colours and chromaticities respectively.

In terms of boundary colour vectors in the SML cone excitation space, the boundary vector pairs which produce the same M and S excitations (for protanomaly) / the same L and S excitations (for deuteranomaly) as the reference colour now have a larger L / M cone excitation difference from the reference colour. The result on the discrimination ellipse is an elongation along the same direction as the corresponding dichromacy case - extending along the protanopic confusion lines for protanomaly or deuteranopic confusion lines for deuteranomaly. However, the elongation is now finite instead of infinite. (Figure 2.20)

The cone shift varies from person to person. Some have little shift while some have large shift. The larger the cone shift, the higher the correlation between the L-cone and M-cone excitations, and thus the larger the discrimination ellipse elongation.

The above discussion explains why in most case, the discrimination ellipse elongations converge to either one of the two copunctal points (Figure 2.21), and have a wide variation on the extent of elongation.



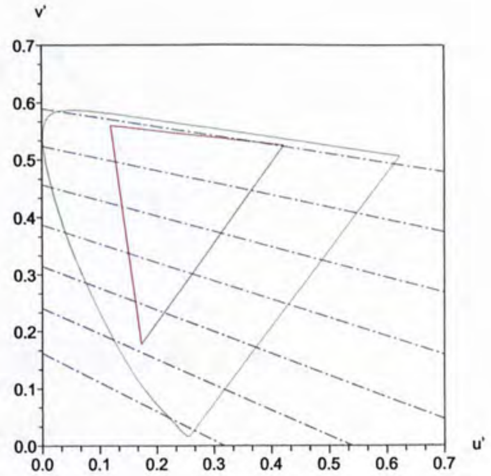


Figure 2.21: Protanopic confusion lines (left) and deuteranopic confusion lines (right). The former converge to the copunctal point (0.67, 0.50) while the latter converge to the copunctal point (-1.22, 0.78).

2.5 Luminance match of colour deficient people

Our knowledge on the biological origin of colour deficiency can also explain the deviated luminance matches of the colour-blind. Figure 2.22 shows the relative luminous efficiency of normal trichromats compared with different kinds of dichromats [2].

While the curve for deuteranopia is similar to that of normal, the curve for protanopia shows a marked reduction in the efficiency at long wavelengths. It is believed to be due to the lack of L-cone to detect light over that region of the visible spectrum. Protanomaly features a continuous range of L-cone shift towards the shorter wavelength. So it can be expected that the corresponding luminous efficiency is some intermediate between those of normal people and protanopic patients [2].

2.6 Diagnosis of colour deficiency

The convergence of discrimination ellipse elongations or confusion lines is useful in finding colours confusing to a patient suffering from a particular kind of colour

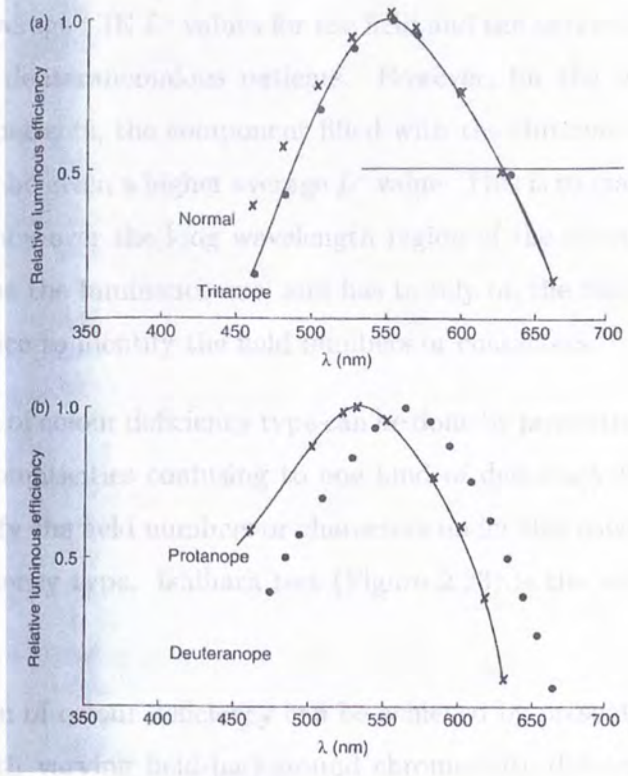


Figure 2.22: Relative luminous efficiency of normal people compared with protanopic, deuteranopic and tritanopic patients. (After Birch [2])

deficiency. Given a colour with chromaticity coordinates (u_c, v_c), the colours that can be confused with (u_c, v_c) lie on the confusion line passing through (u_c, v_c) and the corresponding copunctal point (u_o, v_o). The larger the Euclidean distance between the selected pairs (u_1, v_1) and (u_2, v_2), the larger the chromaticity difference between them.

To eliminate the luminance cue, the confusing chromaticities are usually presented on pseudoisochromatic plates. A pseudoisochromatic plate features a field with numbers or characters over a background. Both field and background are composed of small patches in similar shapes. While the patch chromaticities of the field and background are fixed at (u_1, v_1) and (u_2, v_2) respectively, the luminance levels of the patches are in random.

If the average luminance values of the field and the background are similar (This means similar average CIE L^* values for the field and the background to the deuteranopic and the deuteranomalous patients. However, for the protanopic and the protanomalous patients, the component filled with the chromaticity closer to saturated red should be given a higher average L^* value. This is to match their decreased luminous efficiency over the long wavelength region of the spectrum.), the subject can no longer use the luminance cue, and has to rely on the field-background chromaticity difference to identify the field numbers or characters.

Classification of colour deficiency type can be done by presenting plates with field-background chromaticities confusing to one kind of deficiency but not the others. Failure to identify the field numbers or characters under this condition can conclude the colour deficiency type. Ishihara test (Figure 2.23) is the well-known classic of this kind.

Quantification of colour deficiency can be achieved by presenting pseudoisochromatic plates with varying field-background chromaticity difference. After finding the detection thresholds along different chromatic directions, the discrimination ellipse of the subject about the selected reference chromaticities can be plotted. In fact, the discrimination ellipses shown in Figure 2.13 to 2.16 are produced with an implementation of this kind called Cambridge colour vision test [26]. This colour vision test features the use of a 'C' field character in different orientations, which can be up, down, left or right with equal chances (Figure 2.23). The subject is requested to report the orientations of these plates. Using a field character of this kind has the advantage that a large number of pseudoisochromatic plates can be produced easily. It is in contrast with those using alphanumeric characters. In these cases, if the number of different alphanumeric characters is not large, the subject may get rehearsed in advance and remember the correct responses. This can decrease the creditability of the result significantly [22].

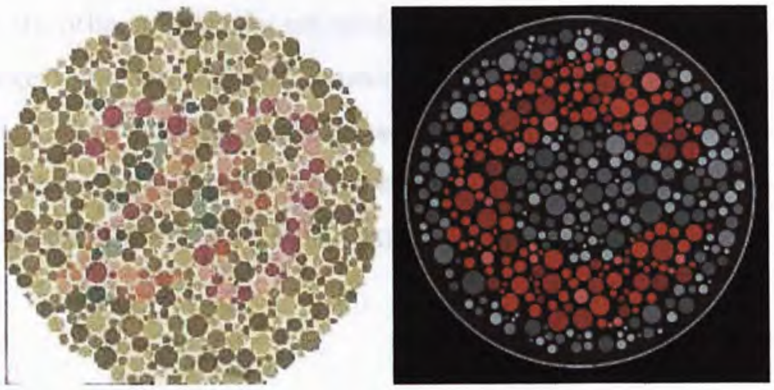


Figure 2.23: Ishihara test plate (left) and Cambridge colour vision test plate (right).

2.7 Dichromat simulation algorithm

The effect of severe colour deficiencies like protanopia and deuteranopia can be directly appreciated by typical trichromats with a dichromat simulation algorithm [3].

The algorithm essentially replaces each set of chromaticities lying on the same confusion line with a single replacement colour. Finding the confusion line on which a colour lies again depends heavily on our knowledge of confusion line convergence. The replacement colour depends on the report made by unilateral dichromats. Unilateral dichromats have one eye suffering from dichromacy while the other eye being totally vision perfect. So they can tell how the colour should be like to typical dichromats by comparing the perceptions from the defective eye and the normal eye.

The dichromatic views so generated tell how images get confused as a result of dichromacy. In fact, the dichromatic views shown in Figures 1.2 and 1.3 are generated with this kind of algorithm. Apart from that practical use, its operation principle does suggest another way to interpret colour deficiency: The algorithm maps several palette entries to the same single colour. This means that colour deficient people have a palette which is a reduced version of that possessed by normal trichromats. In other words, the colour-blind view uses a reduced set of colours to present a larger superset, which is essentially the same as the consequence of colour quantization. Colour quantization tells us that this must incur information loss. A different choice on the reduced palette may decrease the discrimination information loss in some parts of the image, at the expense of increasing the loss in other parts.

Chapter 3

Monocular Compensation

Missed discriminations due to colour deficiency can bring much inconvenience. In this thesis, we propose the use of monocular compensation. This chapter first describes this presentation strategy. A simple demonstration is provided to let the reader appreciate its feasibility and potential benefits.

We regard this presentation strategy as essential in providing missed discriminations while preserving original ones. This is to be explained in the subsequent analysis on other existing solutions not utilizing monocular compensation.

The concept of monocular compensation is not new. Commercial tinted lens systems like X-Chrom [33] and ChromaGen [12] have already adopted this presentation strategy. This chapter also reviews their experience, from which we come up with a few desirable properties of an ideal monocular compensation scheme.

3.1 Principle

The principle of monocular compensation is very simple. While one eye is preserved with the unmodified original view, the other eye is fed with a modified compensated view. The compensated view offers the discriminations missed in the original view.

Figure 3.1: The equipment used for demonstrating monocular compensation: a pair of red-green 3D glasses (left), a four-colour figure (right). The triplets are the RGB values of the patches. These values may need slight adjustments to achieve the effect with your red-green 3D glasses.

Now, both the original and originally missed discriminations are available to the brain. It is likely that missed discriminations can be redeemed.

The feasibility of this presentation strategy can be demonstrated with a pair of red-green 3D glasses and a four-colour figure as shown in Figure 3.1. The 3D glasses can be made with a piece of cardboard, together with some red and green filters. Put on the 3D glasses to view the four-colour figure. Looking through the red filter alone (by closing the eye covered by the green filter), one has a view with the two red patches on the left look the same. Looking through the green filter alone (by closing the eye covered by the red filter), the difference between the two green patches on the right cannot be recognized. However, if the subject views with both eyes, one eye through the red filter while the other eye through the green filter, all the four patches are distinguishable. The proof is now obvious if we regard the red-filter view as the original view and the green-filter view as the compensated view.

Such a complementary relationship is not unidirectional. While the compensated view can assist the original view, the original view can help the compensated view as well. In our example, one cannot find any difference between the green patches in the green-filter view, which we regard as the compensated view. Yet discrimination of these two patches can still be made. It is because the red filter view, which we consider to be the original view, has these two patches show up differently.

3.2 Potential problems without monocular compensation

Monocular compensation is in opposed to binocular compensation, in which the single compensated view is offered to both eyes.

In binocular compensation, the only issue to tackle is the design of the compensation algorithm which generates the compensated view. Intuitively, the best possible compensation algorithm involves figuring out the logical units in the image, and then filling them with sets of non-confusing colours. However, this is too costly to implement, especially in real-time applications. That is why all existing compensation algorithms are context-independent, operating without considering the image context. Below surveys these context-independent algorithms, from which we can see the inadequacy of just having the compensated view.

Stretching chromaticity difference

The discrimination ellipses of colour deficient people feature elongations at some particular chromatic directions. So if the chromaticity differences between the confusing colour pairs are stretched accordingly, degraded discriminations with these pairs can be redeemed.

An immediate question is on how the individual chromaticity is to be changed. The difference between any two confusing chromaticities, despite their locations in the gamut, should all be enlarged to an appropriate extent.

For anomalous trichromats, the reason for their degraded chromatic discrimination (and hence elongated discrimination ellipses) is their distorted cone excitations over all spectra of light. So if we change the chromaticities with the aim of providing them with the original set of excitations or at least excitation ratios, their colour discrimination can get close to normal.

This operation can be achieved by changing the palette of the image display device

[32]. Due to the cone shift, the RGB to SML transformation matrix $T'_{RGB \to SML}$ corresponding to the sensation system of an anomalous trichromat differs from that of normal, $T_{RGB \to SML}$. This approach suggests that the original RGB value $[R \; G \; B]^T$ should be transformed to $[R' \; G' \; B']^T$ by $T_{compensate} = (T'_{RGB \to SML})^{-1} \cdot T_{RGB \to SML}$ before display. Since then, normal cone excitation ratio is back, as shown in Equation 3.1.

$$
\begin{aligned}
\begin{bmatrix} S \\ M \\ L \end{bmatrix} &= T'_{RGB \to SML} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \\
&= T'_{RGB \to SML} \cdot T_{compensate} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \\
&= T'_{RGB \to SML} \cdot (T'_{RGB \to SML})^{-1} \cdot T_{RGB \to SML} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \\
&= T_{RGB \to SML} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \qquad (3.1)
\end{aligned}
$$

The same operation can also be achieved through placing a specially-designed colour filter in front of the image source [1, 17]. Suppose the sensitivity of the anomalous cone is shifted from $\phi(\lambda)$ to $\phi'(\lambda)$. The transfer characteristics of the colour filter $f(\lambda)$ is designed in such a way that $\phi'(\lambda)f(\lambda) = \phi(\lambda)$. Equation 3.2 shows the resulting excitation Φ of the anomalous cone type by a light $e(\lambda)$. It is basically an integral with an integrand being a trifactorial product $\phi'(\lambda)f(\lambda)e(\lambda)$.

$$\Phi = \int_{\lambda} \phi'(\lambda) f(\lambda) e(\lambda) d(\lambda) \qquad (3.2)$$

By embracing the last two factors as in Equation 3.3, the equation provides the physical interpretation of the operation: The light is filtered suitably before sensed

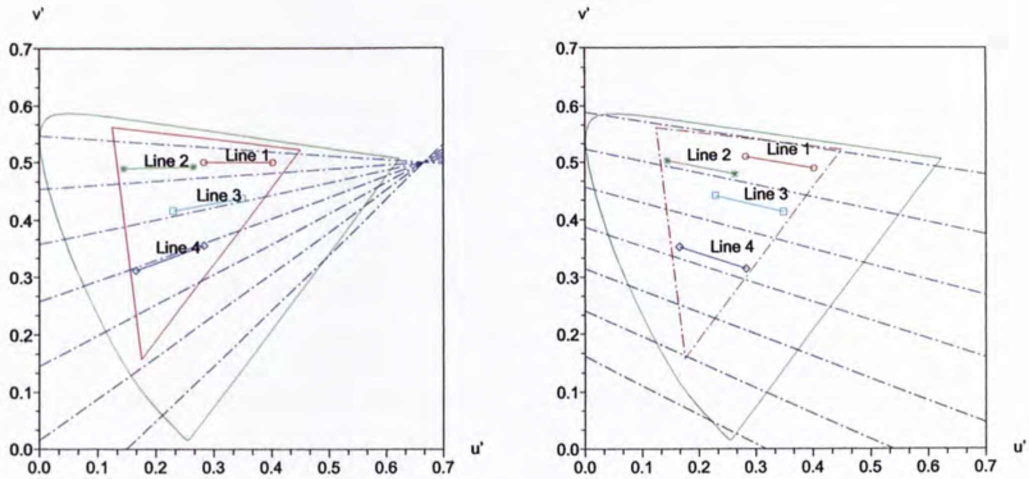


Figure 3.2: The four protanopic (left) and deuteranopic (right) confusing chromaticity pairs for evaluation of colour discrimination improvement scheme in this thesis. All these pairs lie on the corresponding confusion lines, and separated with a $u'v'$ chromaticity difference 0.12. The mid-points (reference chromaticity) of these pairs are as follows: Pair 1 - (0.34, 0.50), Pair 2 - (0.21, 0.49), Pair 3 - (0.29, 0.43), Pair 4 - (0.22, 0.33).

by the anomalous cone.

$$\Phi = \int_\lambda \phi'(\lambda)[f(\lambda)e(\lambda)]d(\lambda) \tag{3.3}$$

Bracketing the first two factors instead as in Equation 3.4, we can see that such a physical filtering is mathematically equivalent to compensating the cone shift.

$$\begin{aligned}\Phi &= \int_\lambda [\phi'(\lambda)f(\lambda)]e(\lambda)d(\lambda) \\ &= \int_\lambda \phi(\lambda)e(\lambda)d(\lambda)\end{aligned} \tag{3.4}$$

Figure 3.2 shows some chromaticity pairs that are confusing to the colour deficient people of different kinds. They are used throughout this thesis for evaluating the effectiveness of colour discrimination improvement strategies.

Figure 3.3 reveals how the chromaticities of these confusing pairs are changed for protanomalous and deuteranomalous patients with different cone shifts. The chromaticity differences of all colours are stretched along the corresponding confusion lines. Each of these stretches compensates for the discrimination ellipse elongation

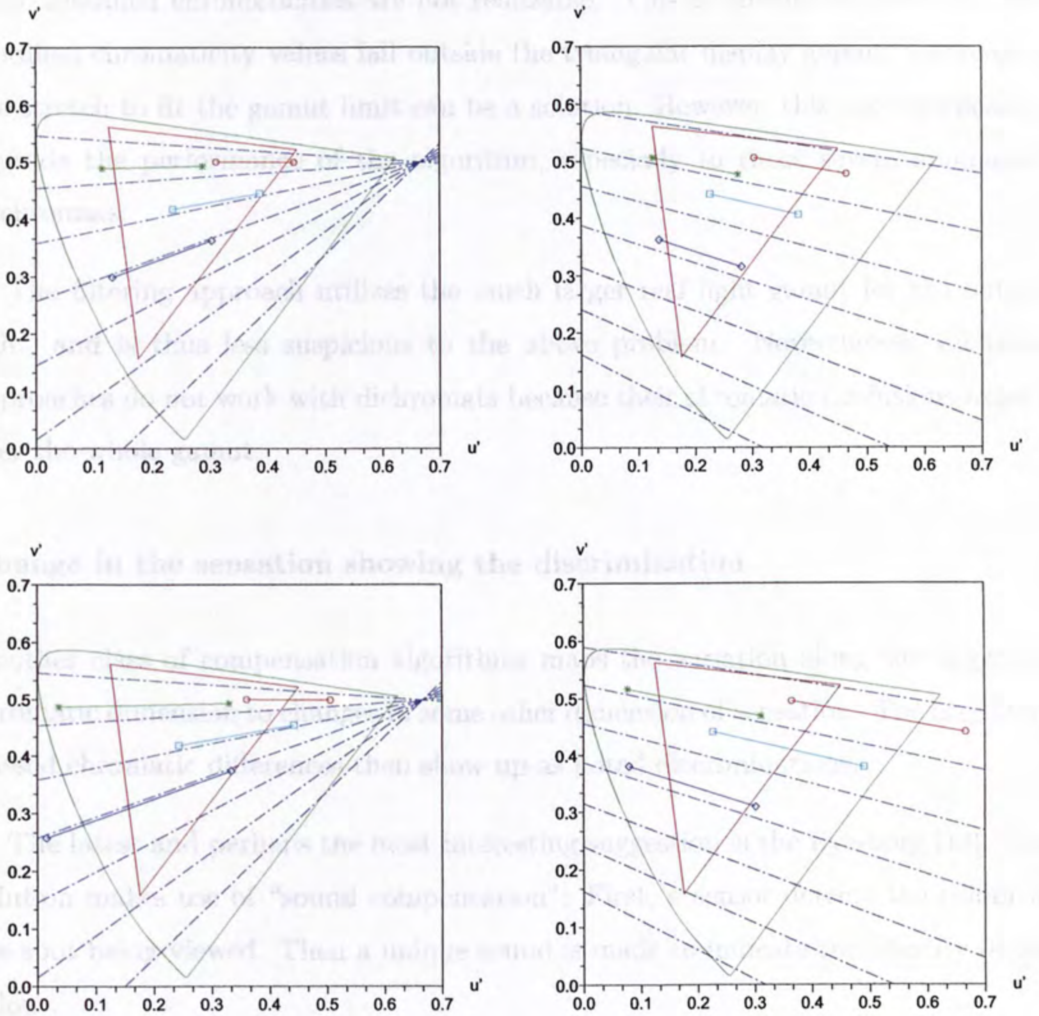


Figure 3.3: The new chromaticities of the protanopic (left) and deuteranopic (right) confusing chromaticity pairs after stretching the chromaticity difference for protanomalous and deuteranomalous patients with different cone shifts. First row: 8nm. Second row: 16nm.

along the same confusion line, thus improving chromatic discrimination.

This approach aims to recover normal cone excitation ratio and hence normal colour vision. So a single compensated view is sufficient to convey all chromatic discrimination information.

However, the palette modification approach suffers from the disadvantage that some modified chromaticities are not realizable. This is obvious as some of these modified chromaticity values fall outside the triangular display gamut. Decreasing the stretch to fit the gamut limit can be a solution. However, this can significantly degrade the performance of the algorithm, especially to those severe anomalous trichromats.

The filtering approach utilizes the much larger real light gamut for the output light, and is thus less suspicious to the above problem. Nevertheless, all these approaches do not work with dichromats because their chromatic confusions extend over the whole gamut.

Change in the sensation showing the discrimination

Another class of compensation algorithms maps the variation along the degraded chromatic dimension to changes in some other dimension of sensation. The originally missed chromatic differences then show up as noted discriminations.

The latest and perhaps the most interesting suggestion is the Eye-borg [13]. The solution makes use of "sound compensation": First, a sensor detects the colour of the spot being viewed. Then a unique sound is made to indicate the identity of the colour.

Other proposals stay with the visual sense. Viewing through tinted filters, for example, presents the missed variations with brightness difference [20]. The originally confusing colour pairs, being filtered differently due to their intrinsic difference in spectral content, are now shown in different brightness for discrimination. Table 3.1 shows how this "luminance compensation" acts to resolve different classes of confusing colours.

Another group of compensation algorithms [6, 25, 28, 32] conveys the missed variations with some other colours non-confusing to the colour-blind. Figure 3.4 plots the new chromaticities of our testing confusing colour pairs (shown in Figure

Class of confusion	*Colour viewed*	*Relative brightness*
red-green	red	darker
	yellow	no change
	green	brighter
purples	reddish-purple	darker
	bluish-purple	lighter
grey-cyan	grey	no change
	blue-green	brighter

Table 3.1: Relative brightness of the confusing colours when viewed through a green filter

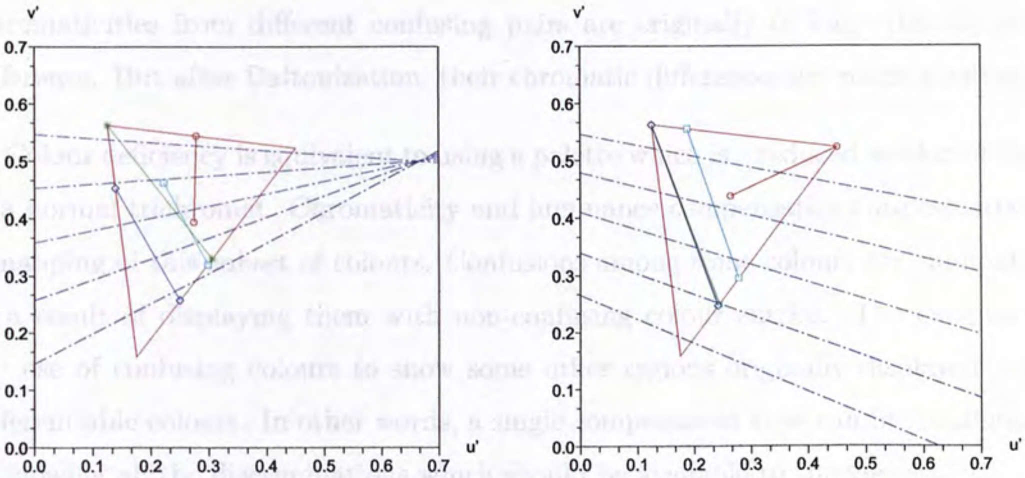


Figure 3.4: The new chromaticities of the protanopic (left) and deuteranopic (right) confusing chromaticity pairs after Daltonization.

3.2) after applying Daltonization [6], a compensation algorithm of this kind. By tilting the displayed chromaticities of the confusing colour pairs from the confusing lines, the originally missed differences now become noticeable. We refer this kind of remedy to be "chromaticity compensation".

This class of algorithms is effective in showing the variations which are originally along the degraded chromatic directions. It is especially so to those severe anomalous trichromats and dichromats, to whom stretching the chromaticity difference along the same confusion line is ineffective.

However, sound compensation offered by the Eye-borg solution does not work in a noisy environment. In the other compensation approaches, new confusions can emerge.

For luminance compensation, a red originally of luminance L_{orig}, though shows a difference from green of luminance L_{orig} after reducing the luminance of the red by ΔL_r and the green by $\Delta L_g \neq \Delta L_r$, may now be confused with some other green which is now of luminance $L_{orig} - \Delta L_r$.

Similar problems in chromaticity compensation can be observed in Figure 3.4. Chromaticities from different confusing pairs are originally of large chromaticity difference. But after Daltonization, their chromatic differences are much smaller.

Colour deficiency is equivalent to using a palette which is a reduced version of that of a normal trichromat. Chromaticity and luminance compensations are essentially remapping of this subset of colours. Confusions among some colours are eliminated as a result of displaying them with non-confusing colour entries. The expense is the use of confusing colours to show some other regions originally displayed with differentiable colours. In other words, a single compensated view can be insufficient in showing all the discriminations which should be available to the viewers.

3.3 Existing monocular compensation implementations

The concept of monocular compensation is not new. X-Chrom [33] in the 1970s and ChromaGen [12] in the 1990s are the two commercial products that have already adopted this presentation strategy to help the colour-blind.

Both systems use a tinted lens to produce a luminance-compensated view for the non-dominant eye. The non-dominant eye is in contrast with the dominant eye which is the eye preferred for sighting a target [24]. The dominant eye can be easily determined as follows: With both eyes opened, use a thumb to block your sight to a distant target. Close each eye in turn, and the eye through which the distant target is still blocked is the dominant eye. The use of a light-absorbing filter inevitably

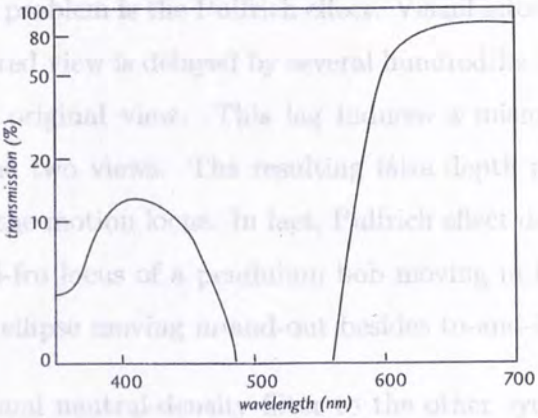


Figure 3.5: X-Chrom lens transmission curve (After McIntyre [20]).

reduces the overall luminance of the luminance-compensated view so generated. By presenting such a view to the non-dominant eye rather than the dominant eye, the viewer is claimed to be less aware of the overall luminance difference between the views, and hence more likely to accept the lens [10, 33].

On the choice of tinted lens, X-Chrom uses a lens with transfer characteristics shown in Figure 3.5 for protanopia, deuteranopia, and all severities of protanomaly and deuteranomaly. ChromaGen, in contrast, suggests a choice of lenses in different colours and intensities. It is believed to adapt to different kinds and severities of colour deficiency [12, 30].

While subjective responses on these lens systems are positive [11, 30], potential undesirable effects have also been pointed out. Many of them are related to the use of light-absorbing filter to generate the compensated view. [20, 27].

The first one is false stereoscopy. Light-absorbing filter fitted over one eye essentially reduces the brightness of all the patches in the compensated view. Because a darker patch in the compensated view spans a smaller retinal size than the brighter counterpart in the original view, binocular differences in the sizes of the patch is induced. False stereoscopy is created as a result.

Another potential problem is the Pulfrich effect. Visual information of the darker luminance-compensated view is delayed by several hundredths of seconds relative to that of the brighter original view. This lag induces a mismatch in positions of moving objects in the two views. The resulting false depth perception can cause misinterpretation of the motion locus. In fact, Pulfrich effect describes the phenomenon that the to-and-fro locus of a pendulum bob moving in front of a viewer can be interpreted as an ellipse moving in-and-out besides to-and-fro.

Fitting an additional neutral-density filter to the other eye has been suggested to counteract the problem. However, this further decreases the light reaching the eyes, thus discouraging the use of the lens system in dark environment [27, 30].

3.4 Compensation algorithm for monocular compensation

To conclude, monocular compensation presentation strategy brings both opportunities and constraints to the underlying compensation algorithm.

Under this presentation strategy, we have the original view supplying the original discriminations. So the compensated view can concentrate on redeeming the missed difference. However, such necessary changes in the compensated view inevitably introduce unfamiliar binocular dissimilarity. In fact, our experience of building stereoscopic applications tells us that too large a dissimilarity is uncomfortable to the viewers. The larger the dissimilarity, the larger the discomfort. So the difference between the two views should be kept as minimum as possible. After learning how the potential problems of X-Chrom and ChromaGen emerge, we know we should keep the luminance difference between the two views as small as possible in particular.

Meanwhile, as suggested by the variety of tinted lenses in ChromaGen system, the compensation algorithm may have some flexibility to adapt to a variety of colour deficiency types and severities.

Chapter 4

Stereo Visual Display Unit - Monocular Compensation

With the rapid development of multimedia and telecommunication technologies, the human visual world is now filled with images reproduced with VDUs like CRTs or LCDs. Any handy strategy that improves the colour vision experience of the colour-blind over these devices can thus help a lot.

"Stereo Visual Display Unit - Monocular Compensation", or SVDU-MC for short, is our colour vision improvement candidate for use with VDUs. It targets at the majority of colour-blind population, including those suffering from protanopia, protanomaly, deuteranopia and deuteranomaly.

SVDU-MC implements monocular compensation presentation strategy, and has the stereoscopic display capability of the VDU as the only hardware requirement. While the method to feed the views is trivial with stereoscopy, the compensation algorithm is much more complicated. Gamut-based palette compression, or GBPC for short, has been designed to avoid unnecessary colour changes especially in the luminance domain. We have divided the compensation algorithm into three blocks as shown in Figure 4.1, they are discussed one by one in the subsequent sections.

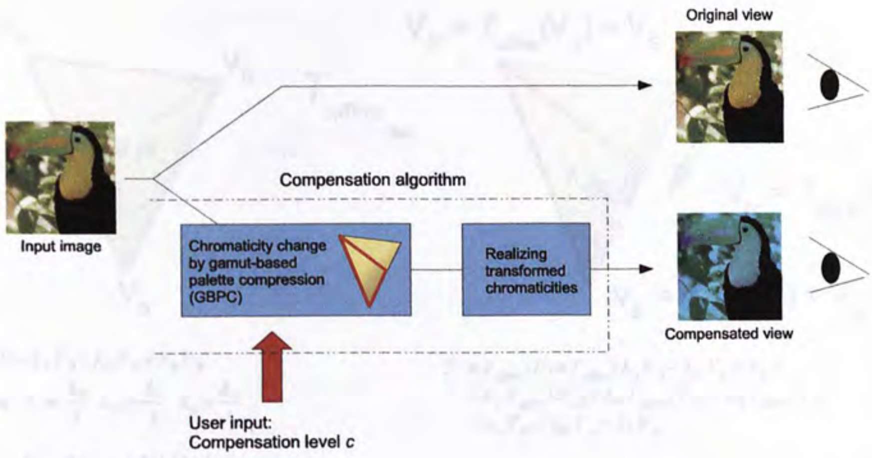


Figure 4.1: System overview of Stereo Visual Display Unit - Monocular Compensation (SVDU-MC)

4.1 Gamut-based palette compression

To minimize unnecessary binocular luminance difference, we have selected chromaticity compensation instead of luminance compensation for SVDU-MC. Figures 4.2 summarizes the operation of our chromaticity compensation algorithm which we name "gamut-based palette compression", or GBPC for short.

Our algorithm stems from the following observation: The set of protanopic and deuteranopic confusing colours are aligned approximately in parallel to the R-G side of the gamut. So if we display these confusing colours with some other linear series tilted from the original sets, discrimination among them can be improved.

GBPC suggests a geometric compression of the VDU display gamut in a perceptually uniform chromaticity space like CIE $u'v'$ (1976). With the chromaticity coordinates of the VDU red primary vertex, green primary vertex and blue primary vertex denoted as V_R, V_G and V_B, the compression is initiated by shortening the length V_RV_B about the fixed point V_B. As a result, the gamut is changed from $V_RV_GV_B$ to $V_{R'}V_GV_B$ where $V_{R'}$ refers to the chromaticity of some particular mix of red and blue. Within a set of confusing colours, we expect those close to the

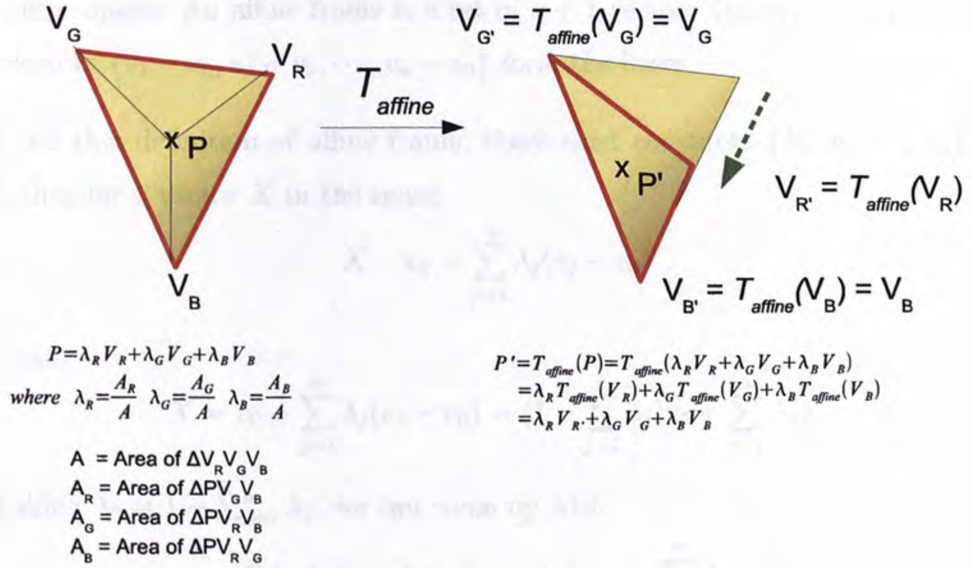


Figure 4.2: Operation of gamut-based palette compression (GBPC)

red-blue boundary are pushed towards the blue vertex. The confusing colour series are thus tilted from the confusion lines.

To implement this, the associated chromaticity transformation function $T : \Re^2 \rightarrow \Re^2$ needs to preserve the relative positions of the colours in the compressed gamut: For a chromaticity (u_0, v_0) which divides the line segment bounded by some two chromaticities (u_1, v_1), (u_2, v_2) in some ratio $a : b$, we expect its transformed value (u_0', v_0') to divide the transformed line segment bounded by (u_1', v_1') and (u_2', v_2') by the same ratio. This condition can be satisfied if the transformation T is affine. A transformation $T : \Re^n \rightarrow \Re^n$ is affine if

$$T((1-t)P + tQ) = (1-t)T(P) + tT(Q) \tag{4.1}$$

for $P, Q \in \Re^n$ and $t \in \Re$. In GBPC which operates on the gamut triangle, such an affine transformation T_{affine} to find the new chromaticity coordinates P' of a chromaticity P can be easily evaluated with barycentric coordinates.

A set of barycentric coordinates $(\lambda_0, \lambda_1, \cdots, \lambda_n)$ is the unique characterization of a point P in an affine space $\Re^n$. They are related to affine frames associated with the affine space: An affine frame is a set of $n+1$ vectors $\{v_0, v_1, \cdots, v_n\}$ such that the vectors $\{v_1 - v_0, v_2 - v_0, \cdots, v_n - v_0\}$ form the bases.

From this definition of affine frame, there exist constants $\{\lambda_1, \lambda_2, \cdots, \lambda_n\} \in \Re$ such that for a vector X in the space

$$X - v_0 = \sum_{j=1}^{n} \lambda_j (v_j - v_0) \tag{4.2}$$

Hence,

$$X = v_0 + \sum_{j=1}^{n} \lambda_j (v_j - v_0) = (1 - \sum_{j=1}^{n} \lambda_j) v_0 + \sum_{j=1}^{n} \lambda_j v_j \tag{4.3}$$

Taking $\lambda_0 = 1 - \sum_{j=1}^{n} \lambda_j$, we can come up with

$$X = \lambda_0 v_0 + \lambda_1 v_1 + \cdots + \lambda_n v_n = \sum_{j=0}^{n} \lambda_j v_j \tag{4.4}$$

with $\sum_{j=0}^{n} \lambda_j = 1$.

As a result, these λ_j's are uniquely determined by the vector X. These λ_j are the barycentric coordinates mentioned above.

The barycentric coordinates have some physical meaning: The λ_j's correspond to the masses placed at the respective position $v_j's$ such that X is the geometric centroid of the masses.

With the VDU gamut triangle which is in $\Re^2$, the vectors $\{v_0, v_1, v_2\}$ comprising the affine frame are simply the gamut triangle vertices, each corresponding to the chromaticity coordinates of a colour primary. If we denote the vectors corresponding to the red, green and blue primary chromaticities and the respective barycentric coordinates as $\{V_R, V_G, V_B\}$ and $\{\lambda_R, \lambda_G, \lambda_B\}$ respectively, a chromaticity P in the gamut can be represented as

$$P = \lambda_R V_R + \lambda_G V_G + \lambda_B V_B \tag{4.5}$$

with $\lambda_R + \lambda_G + \lambda_B = 1$

To obtain the transformed chromaticity P' of a chromaticity P using barycentric coordinates, two steps are involved:

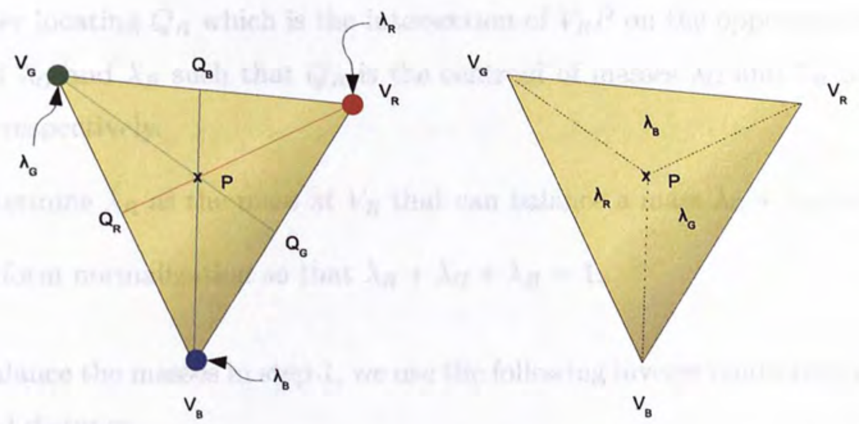


Figure 4.3: Physical meaning of barycentric coordinates of a point P in a triangle. Left: They are the masses located at the respective vertices so the point P is the geometric centroid of the masses. Right: They are in proportion to the triangles split by P.

1. Calculate the barycentric coordinates of P
2. Evaluate the transformed chromaticity P'.

Below shows how these two tasks can be easily implemented with the properties of barycentric coordinates and affine transform [4, 9].

Step 1 *The barycentric coordinates $(\lambda_R, \lambda_G, \lambda_B)$ of a point P in the triangular VDU gamut can be found with area ratios.*

$$\lambda_R = \frac{A_R}{A} \qquad \lambda_G = \frac{A_G}{A} \qquad \lambda_B = \frac{A_B}{A}$$

where A, A_R, A_G and A_B are the areas of $\Delta V_R V_G V_B$, $\Delta P V_G V_B$, $\Delta P V_R V_B$ and $\Delta P V_R V_G$ respectively.

Figure 4.3 provides a summary of what are to be discussed here.

To find the values of λ_R, λ_G and λ_R, one can make use of their physical meaning: They correspond to some three masses placed at V_R, V_G and V_B respectively such that P is the geometric centroid of the three masses. The formal way to find the coordinates from this first principle is as follows:

1. After locating Q_R which is the intersection of V_RP on the opposite side V_GV_B, find λ_G and λ_B such that Q_R is the centroid of masses λ_G and λ_B at V_G and V_B respectively.

2. Determine λ_R as the mass at V_R that can balance a mass $\lambda_G + \lambda_B$ at Q_R.

3. Perform normalization so that $\lambda_R + \lambda_G + \lambda_B = 1$.

To balance the masses in step 1, we use the following inverse relationship between mass and distance.

$$\frac{\lambda_G}{\lambda_B} = \frac{V_BQ_R}{Q_RV_G} \tag{4.6}$$

Which two masses to balance first is arbitrary. Thus we can come up with the following two equations.

$$\frac{\lambda_R}{\lambda_B} = \frac{V_BQ_G}{Q_GV_R} \tag{4.7}$$

$$\frac{\lambda_G}{\lambda_R} = \frac{V_RQ_B}{Q_BV_G} \tag{4.8}$$

From the three ratios given in Equations 4.6 to 4.8, the λ_j's can be obtained.

To further simplify the operation, note that the areas of the triangles ΔPV_RV_B, ΔPV_GV_B and ΔPV_BV_R are proportional to the barycentric coordinates. For example, if we derive further from Equation 4.6

$$\begin{aligned}\frac{\lambda_G}{\lambda_B} &= \frac{V_BQ_R}{Q_RV_G} = \frac{Area_{\Delta V_RV_BQ_R}}{Area_{\Delta V_RQ_RV_G}} \\ &= \frac{Area_{\Delta PV_BQ_R}}{Area_{\Delta PQ_RV_G}} = \frac{Area_{\Delta V_RV_BQ_R} - Area_{\Delta PV_BQ_R}}{Area_{\Delta V_RQ_RV_G} - Area_{\Delta PQ_RV_G}} \\ &= \frac{Area_{\Delta PV_RV_B}}{Area_{\Delta PV_GV_R}}\end{aligned} \tag{4.9}$$

Similar derivations for Equations 4.7 and 4.8 are as follows.

$$\frac{\lambda_R}{\lambda_B} = \frac{Area_{\Delta PV_GV_B}}{Area_{\Delta PV_GV_R}} \tag{4.10}$$

$$\frac{\lambda_G}{\lambda_R} = \frac{Area_{\Delta PV_BV_R}}{Area_{\Delta PV_BV_G}} \tag{4.11}$$

Denote A, A_R, A_G and A_B as the areas of $\Delta V_RV_GV_B$, ΔPV_GV_B, ΔPV_RV_B and ΔPV_RV_G respectively, the λ_j's, says λ_R, can be found with area ratios of the triangles as follows.

$$\begin{aligned}\frac{A}{A_R} &= \frac{A_R + A_G + A_B}{A_R} = 1 + \frac{A_G}{A_R} + \frac{A_B}{A_R}\\ &= 1 + \frac{\lambda_G}{\lambda_R} + \frac{\lambda_B}{\lambda_R} = \frac{\lambda_R + \lambda_G + \lambda_B}{\lambda_R} = \frac{1}{\lambda_R}\\ &\Rightarrow \lambda_R = \frac{A_R}{A}\end{aligned} \tag{4.12}$$

With similar deviations, we obtain the following equations:

$$\lambda_G = \frac{A_G}{A} \tag{4.13}$$

$$\lambda_B = \frac{A_B}{A} \tag{4.14}$$

Step 2 *With the barycentric coordinates $(\lambda_R, \lambda_G, \lambda_B)$ of the chromaticity P in the original gamut $V_RV_GV_B$ in hand, the new chromaticity coordinates P' in the compressed gamut $V_{R'}V_GV_B$ after the affine transformation T_{affine} can be calculated as*

$$P' = T_{affine}(P) = \lambda_R V_{R'} + \lambda_G V_G + \lambda_B V_B$$

where $T_{affine}(V_R) = V_{R'}$, $T_{affine}(V_G) = V_G$ and $T_{affine}(V_B) = V_B$.

An affine transformation T_{affine} is in fact the composition of a linear transformation T_{linear} and a translation. In particular, consider a transformation $T(u) = T_{affine}(u) - T_{affine}(0)$ for any $u \in \Re^n$. T is a linear transformation, as proved with Equations 4.15 and 4.16.

$$\begin{aligned}T(\lambda u) &= T_{affine}(\lambda u) - T_{affine}(0)\\ &= T_{affine}(\lambda u + (1-\lambda)0) - T_{affine}(0)\\ &= \lambda T_{affine}(u) + (1-\lambda)T_{affine}(0) - T_{affine}(0)\end{aligned}$$

$$
\begin{aligned}
&= \lambda T_{affine}(u) - \lambda T_{affine}(0) \\
&= \lambda(T_{affine}(u) - T_{affine}(0)) \\
&= \lambda T(u)
\end{aligned} \tag{4.15}
$$

$$
\begin{aligned}
T(u+v) &= T(2 \cdot \frac{u+v}{2}) \\
&= 2 \cdot T(\frac{u+v}{2}) \\
&= 2 \cdot T_{affine}(\frac{u+v}{2}) - 2 \cdot T_{affine}(0) \\
&= 2 \cdot \frac{1}{2} \cdot T_{affine}(u) + 2 \cdot \frac{1}{2} \cdot T_{affine}(v) - 2T_{affine}(0) \\
&= T_{affine}(u) - T_{affine}(0) + T_{affine}(v) - T_{affine}(0) \\
&= T(u) + T(v)
\end{aligned} \tag{4.16}
$$

Denote the linear transformation T as T_{linear}, we have

$$
T_{affine}(u) = T_{linear}(u) + T_{affine}(0) \tag{4.17}
$$

As a result,

$$
\begin{aligned}
P' &= T_{affine}(P) = T_{affine}(\lambda_R V_R + \lambda_G V_G + \lambda_B V_B) \\
&= T_{linear}(\lambda_R V_R + \lambda_G V_G + \lambda_B V_B) + T_{affine}(0) \\
&= \lambda_R T_{linear}(V_R) + \lambda_G T_{linear}(V_G) + \lambda_B T_{linear}(V_B) + T_{affine}(0) \\
&= \lambda_R [T_{affine}(V_R) - T_{affine}(0)] + \lambda_G [T_{affine}(V_G) - T_{affine}(0)] \\
&\quad + \lambda_B [T_{affine}(V_B) + T_{affine}(0)] + T_{affine}(0) \\
&= \lambda_R T_{affine}(V_R) + \lambda_G T_{affine}(V_G) + \lambda_B T_{affine}(V_B) + [1 - (\lambda_R + \lambda_G + \lambda_B) T_{affine}(0)] \\
&= \lambda_R T_{affine}(V_R) + \lambda_G T_{affine}(V_G) + \lambda_B T_{affine}(V_B) + (1-1) T_{affine}(0) \\
&= \lambda_R T_{affine}(V_R) + \lambda_G T_{affine}(V_G) + \lambda_B T_{affine}(V_B) \\
&= \lambda_R V_{R'} + \lambda_G V_G + \lambda_B V_B
\end{aligned} \tag{4.18}
$$

The mapping of any chromaticity $P \to P'$ is determined solely by the compression ratio $c = 1 - V_{R'}V_B / V_R V_B$. Also, the mapping is from a set of colours to a smaller subset whose size is governed by c. This is equivalent to a reduction in the colourfulness, or the palette, of the compensated view. So the compensation view is essentially a display of the original view with a reduced palette, whose entries are determined according to a geometric compression of the display gamut. This algorithm is so named to reflect these characteristics.

Figures 4.4 to 4.5 show how the lines joining sets of confusing colours are tilted from the confusion lines at different compensation levels c.

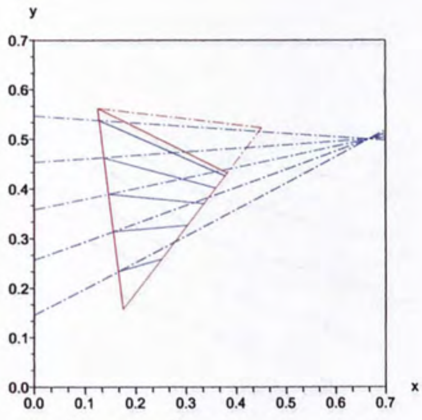
Among the same set of confusing colours, there are some colours (those close to the $V_G V_B$ side of the gamut) which are relatively unaltered, while some others (those close to the $V_R V_B$ side of the gamut) act the opposite. This seems to be the minimal colour change that a context-independent algorithm can achieve, while offering noticeable variations among the confusing colours.

4.2 Compensation level

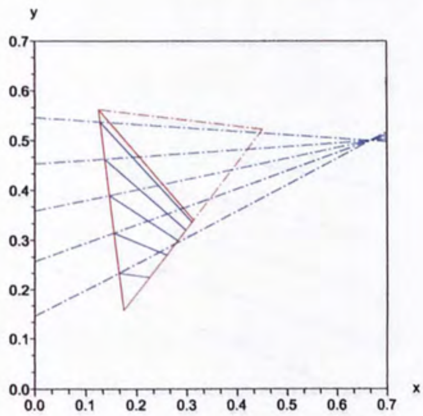
As c increases, the display colours of series of confusing colours are tilted further and further from the confusion lines. While other factors like the Euclidean distance spanned by the confusing pairs are held relatively constant, the result is an improvement in discrimination among these colours. Thus c can be regarded as the compensation parameter.

The question then comes to the determination of a suitable c for a particular colour-blind person.

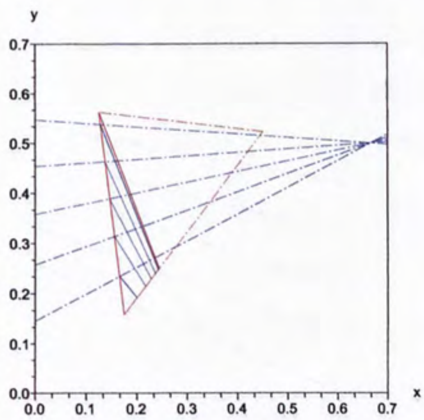
Figure 4.6 shows the discrimination ellipses of people with different severities of colour deficiency. To allow the detection of some two confusing colour pairs distant 1 JND apart, the line joining the confusing colours needs to be tilted $90°$ from the confusion line. This is equivalent to a very large compensation level c, as large as 0.75 for the protanopic and protanomalous patients.



θ	*Angle tilted*	*Distance spanned*
$-4°$	$20°$ to $38°$	0.27 (0.32)
$4°$	$19°$ to $31°$	0.23 (0.28)
$12°$	$18°$ to $27°$	0.19 (0.13)
$20°$	$16°$ to $22°$	0.15 (0.07)
$28°$	$12°$ to $14°$	0.09 (0.07)

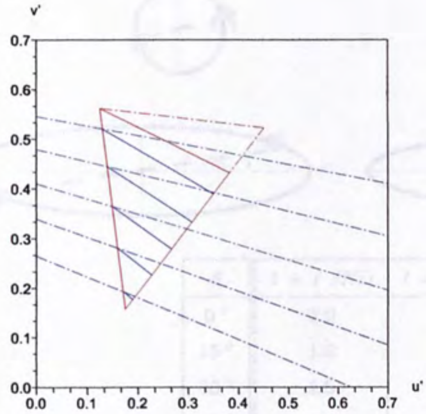


θ	*Angle tilted*	*Distance spanned*
$-4°$	$44°$ to $71°$	0.27 (0.32)
$4°$	$46°$ to $67°$	0.21 (0.28)
$12°$	$46°$ to $62°$	0.16 (0.13)
$20°$	$43°$ to $52°$	0.11 (0.07)
$28°$	$36°$ to $40°$	0.06 (0.07)

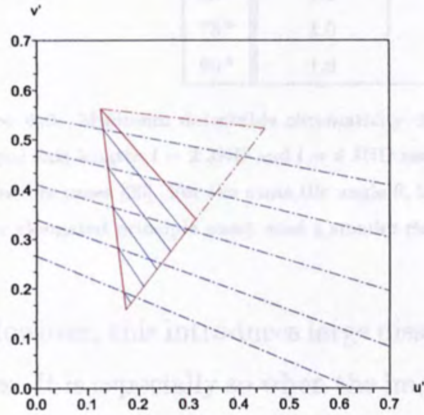


θ	*Angle tilted*	*Distance spanned*
$-4°$	$64°$ to $98°$	0.31 (0.32)
$4°$	$70°$ to $91°$	0.24 (0.28)
$12°$	$74°$ to $91°$	0.18 (0.13)
$20°$	$78°$ to $90°$	0.12 (0.07)
$28°$	$78°$ to $84°$	0.05 (0.07)

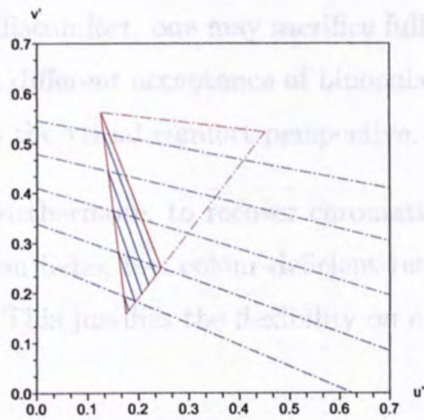
Figure 4.4: Tilting of the protanopic confusing colours (originally lying on the blue dash-dot protanopic confusion lines with slope $tan\theta$, now lying on the blue solid line) at different GBPC c values: (From top to bottom) 0.25, 0.50, 0.75. The associated tables show the numerical values of the tilting from the confusion lines, and the Euclidean distance spanned by the most dissimilar chromaticity pair in each confusing set. The original distance before compensation is included in bracket.



θ	*Angle tilted*	*Distance spanned*
$-11°$	$17°$ to $20°$	0.26 (0.28)
$-14°$	$18°$ to $20°$	0.20 (0.22)
$-17°$	$19°$ to $20°$	0.15 (0.15)
$-20°$	$17°$ to $18°$	0.09 (0.09)

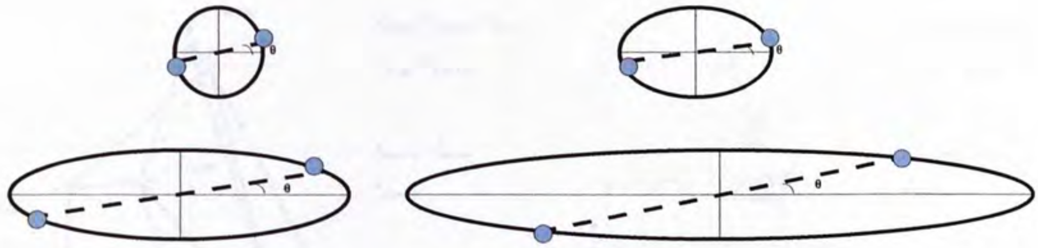


θ	*Angle tilted*	*Distance spanned*
$-11°$	$31°$ to $41°$	0.26 (0.28)
$-14°$	$35°$ to $40°$	0.21 (0.22)
$-17°$	$35°$ to $38°$	0.15 (0.15)
$-20°$	$36°$ to $37°$	0.09 (0.09)



θ	*Angle tilted*	*Distance spanned*
$-11°$	$49°$ to $59°$	0.30 (0.28)
$-14°$	$49°$ to $56°$	0.24 (0.22)
$-17°$	$49°$ to $54°$	0.18 (0.15)
$-20°$	$48°$ to $48°$	0.11 (0.09)

Figure 4.5: Tilting of the deuteranopic confusing colours (originally lying on the blue dash-dot deuteranopic confusion lines with slope $tan\theta$, now lying on the blue solid line) at different GBPC c values: (From top to bottom) 0.25, 0.50, 0.75. The associated tables show the numerical values of the tilting from the confusion lines, and the Euclidean distance spanned by the most dissimilar chromaticity pair among each confusing set. The original distance before compensation is included in bracket.



θ	l = 1 JND	l = 2 JND	l = 4 JND	l = 8 JND
0°	1.0	2.0	4.0	8.0
15°	1.0	1.8	3.5	3.8
30°	1.0	1.8	2.0	2.0
45°	1.0	1.4	1.4	1.4
60°	1.0	1.1	1.2	1.2
75°	1.0	1.0	1.0	1.0
90°	1.0	1.0	1.0	1.0

Figure 4.6: Minimum detectable chromaticity difference for colour deficient people of different severities. The principal axis lengths $l = 2$ JND and $l = 4$ JND resemble those of mild colour deficiency while $l = 8$ JND represents those severe cases [26]. For the same tile angle θ, less severe patients (with discrimination ellipses having relatively shorter elongated principle axes) need a smaller chromaticity difference for discrimination.

However, this introduces large dissimilarity between the original and compensated views. It is especially so when the image is rich of colours in the red region, because it is those red colours that suffer from large chromaticity changes in GBPC. To reduce the discomfort, one may sacrifice full discrimination recovery. Different people may have different acceptance of binocular dissimilarity, so justifying the flexibility on c from the visual comfort perspective.

Furthermore, to recover chromatic discrimination to the same partial level, the person being less colour-deficient requires relatively less tilting as shown in Figure 4.6. This justifies the flexibility on c from the deficiency severity perspective.

4.3 Realizing transformed chromaticities

Colours, after having their chromaticities transformed by GBPC, are displayed on the compensated view with luminance unmodified by default. This is to avoid

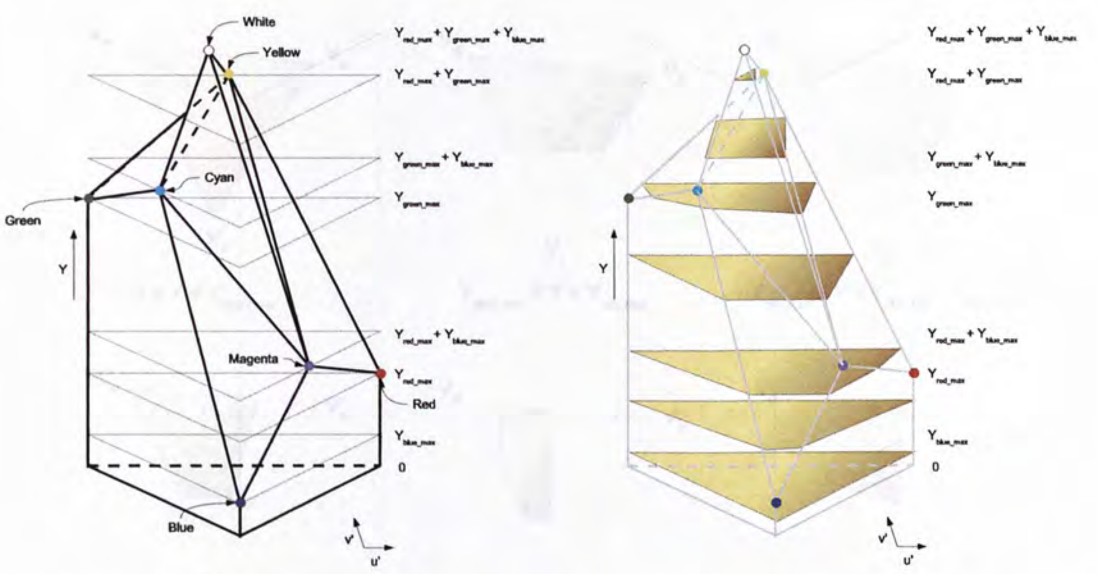


Figure 4.7: Colours (in $Yu'v'$ colour space) defined for a typical VDU (left) and chromaticities defined at different luminance levels (right). The 3D colour gamut is plotted with CIE Y value as the vertical axis and the $u'v'$ grid as the cross-sectional plane. Y_{red_max}, Y_{green_max} and Y_{blue_max} are the maximum possible CIE Y value that can be achieved by the red, green and blue primary alone respectively.

undesirable binocular luminance dissimilarity which has been shown problematic from X-Chrom and ChromaGen experience.

However, this practice can fail in some cases. The cause becomes apparent when we look into Figure 4.7 showing the set of colours realizable by a VDU in a 3D colour space. The colour space is plotted with luminance as the vertical axis, and the $u'v'$ grid as the cross-section plane. Denote the luminance values of the red, green and blue primaries when they operate at full intensity as Y_{red_max}, Y_{green_max} and Y_{blue_max} respectively. There exists considerable differences among these three values. In particular, $Y_{green\_max} > Y_{red\_max} > Y_{blue\_max}$. The consequence is that the cross-section of the 3D VDU gamut is not a full triangle at high luminance. Certain colours are not available at a particular luminance level. For example, at a luminance slightly bigger than Y_{blue_max}, one cannot have it achieved with the blue primary only. Powers contributed by other primaries are needed. In other words, saturated blue is not realizable at that luminance. Subsequently, the gamut is no longer a complete triangle at high luminance. Figure 4.8 provides an alternative

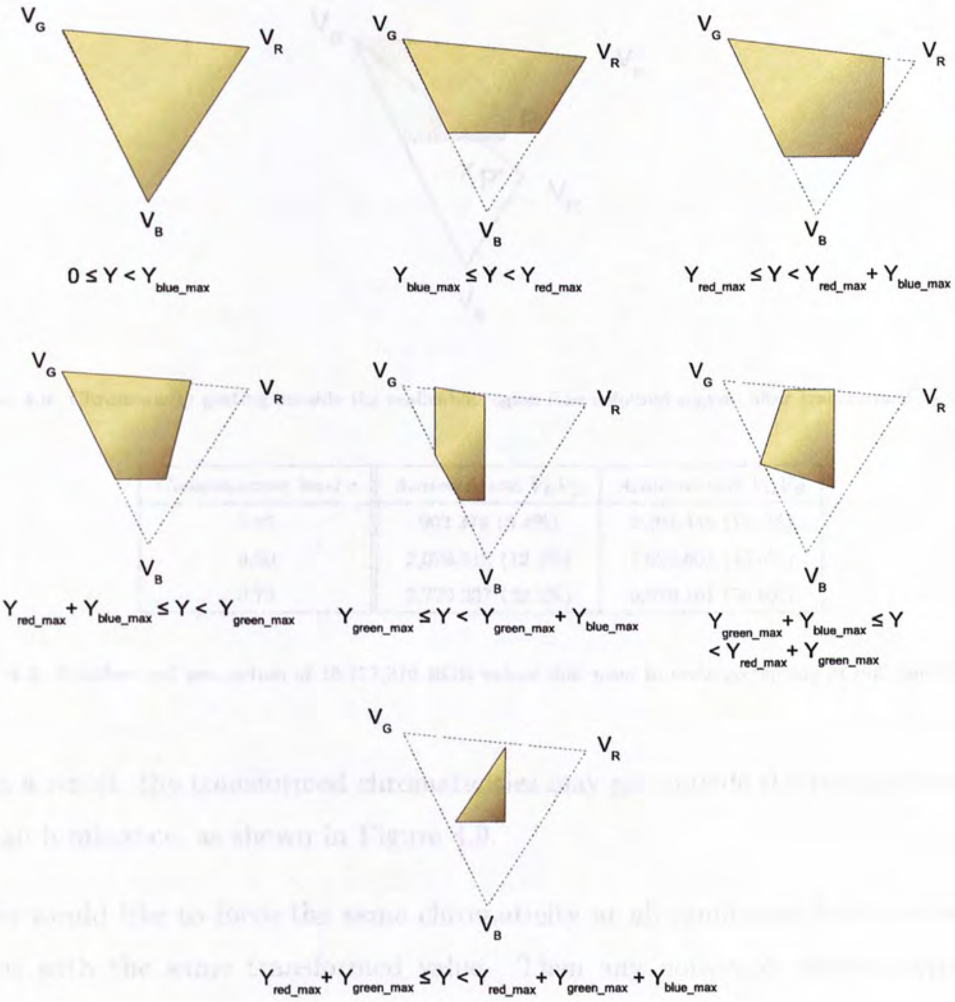


Figure 4.8: Realizable $u'v'$ gamut (the coloured regions) at different luminance values (in CIE Y values). Y_{red_max}, Y_{green_max} and Y_{blue_max} are the maximum possible CIE Y value that can be achieved by the red, green and blue primary alone respectively.

view of such gamut shape irregularities.

In the early days, designers of computer displays decided that equal R, G and B digital codes should produce a neutral chromaticity (with zero saturation). The maximum intensities (and hence luminance values) of the colour primaries are thus set as such, so that the maximum RGB value which corresponds to all the three primaries operating at maximal intensities gives the brightest white [23].

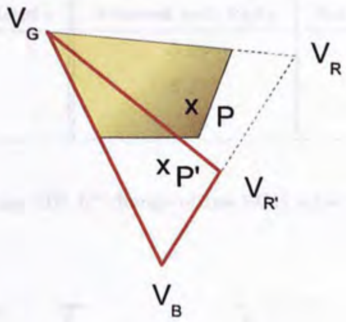


Figure 4.9: Chromaticity getting outside the realizable region (the coloured region) after transformed by GBPC.

Compensation level c	*Achieved with $V_R V_B$*	*Achieved with $V_G V_B$*
0.25	902,378 (5.4%)	3,201,446 (19.1%)
0.50	2,029,813 (12.1%)	7,626,603 (45.5%)
0.75	3,729,237 (22.2%)	9,870,161 (58.8%)

Table 4.1: Number and proportion of 16,777,216 RGB values that need to undergo remedy at different GBPC c values.

As a result, the transformed chromaticities may get outside the realizable region at high luminance, as shown in Figure 4.9.

We would like to force the same chromaticity at all luminance levels to be displayed with the same transformed value. Then any noticeable chromaticity difference observed in the compensated view can be attributed to the intrinsic chromaticity difference of the patches concerned. It can easily be achieved using the maximum possible luminance with these problematic transformed chromaticities. In these cases, luminance change is unavoidable.

Other remedies are possible. But the result should not be better than when no remedy is applied. So we should try to keep the number of remedy cases as small as possible.

In fact, shortening $V_G V_B$ about V_B as in Figure 4.10 was once in our possible choices in the design stage of GBPC. When compared with our current implementation which shortens $V_R V_B$ about V_B, this alternative can also offer similar tilting

Compensation level c	*Achieved with $V_R V_B$*	*Achieved with $V_G V_B$*
0.25	-3.49	-7.06
0.50	-5.41	-13.91
0.75	-7.43	-21.44

Table 4.2: Average CIE L^* change of the RGB values that need remedy.

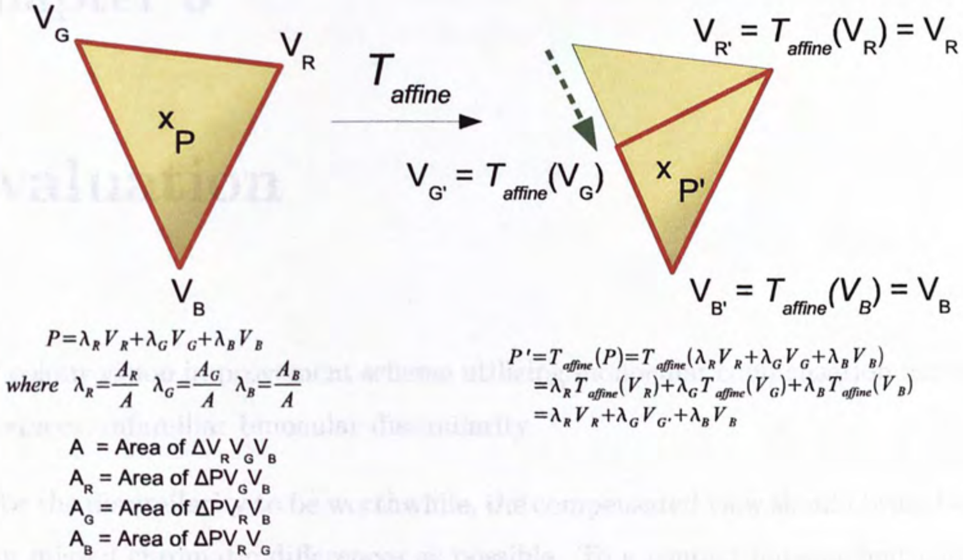


Figure 4.10: Alternative GBPC operation: compressing $V_G V_B$ about V_B

effect to the sets of confusing colours aligning approximately in parallel to the gamut side $V_R V_G$. However, this approach requires more palette entries to undergo the remedy (Table 4.1). The average luminance drop per change is also larger (Table 4.2). This option is eventually abandoned.

Chapter 5

Evaluation

Any colour vision improvement scheme utilizing monocular compensation inevitably introduces unfamiliar binocular dissimilarity.

For the dissimilarity to be worthwhile, the compensated view should bring back as many missed chromatic differences as possible. To a context-independent compensation algorithm of this kind, this means it should work equally well with confusing colours throughout the gamut. The compensated view should also cooperate nicely with the original view, so that discriminations in the two views are well combined. To be accepted for use, the improvement strategy should demonstrate useful discrimination improvements on viewing real-life images. Meanwhile, the associated visual comfort should be of an acceptable level.

This chapter evaluates SVDU-MC using the above criteria. Three experiments have been conducted with five colour deficient people and five normal trichromats. They are all students in The Chinese University of Hong Kong, and are all recruited by personal contact. Table 5.1 summarizes the diagnosis information, including their colour deficiency type and dominant eye. The colour deficiency type was found with Ishihara test shown in Chapter 2, while the dominant eye was figured out with the focusing task described in the same chapter.

Subject	*Classification*	*Dominant eye*
WHS	N	left
KS	N	left
SW	N	right
TW	N	left
DH	N	right

Subject	*Classification*	*Dominant eye*
WH	P	left
KM	P	left
CP	P	left
SK	D	left
KH	D	left

Table 5.1: Colour deficiency diagnosis and dominant eyes of the subjects. Left: normal group. Right: colour deficient group. In the "Classification" column, N refers to normal. P refers to protanopia or protanomaly. D refers to deuteranopia or deuteranomaly.

5.1 Extensiveness of compensation

Objective

To investigate whether GBPC can recover chromatic discrimination over different gamut regions.

Stimuli

Depending on the deficiency type of the subject, 16 protanopic or deuteranopic chromaticity pairs were calculated according to the (reference chromaticity, chromaticity difference) combinations shown in Table 5.2. These pairs were aligned on the lines shown in Figure 3.2. The two chromaticities in each pair were used for filling respectively the fields and the backgrounds of four pseudoisochromatic plates, each featuring a 'C' field character in some orientation (up, down, left or right).

	$u'v'$ chromaticity difference			
Reference chromaticity	0.03	0.06	0.09	0.12
Line 1 (0.34, 0.50)				
Line 2 (0.21, 0.49)				
Line 3 (0.29, 0.43)				
Line 4 (0.22, 0.33)				

Table 5.2: Matrix showing the (reference chromaticity, chromaticity difference) combinations used in experiment 5.1.

Procedure

Under typical room lighting condition, the 64 pseudoisochromatic plates were presented, four at a time in the form of a 2×2 array at a particular GBPC compensation level c, on a CRT with stereoscopic display capability. Each pseudoisochromatic plate spanned a size of $10cm \times 10cm$ on our 17 inch monitor.

The subject, sitting at 80cm away, viewed the plates through the stereoscope which presented the compensated view to both eyes. He was requested to report the 'C' character orientation of each pseudoisochromatic plate (up, down, left, right or failed).

For each GBPC $c = 0.0, 0.25, 0.50, 0.75$, the number of correct recognitions (0 to 4) corresponding to each (reference chromaticity, chrominance difference) combination was recorded.

Result and analysis

Tables 5.3 and 5.4 show the recognition results of the five colour deficient subjects at different compensation levels c. Each table is accompanied with the recognition result of a colour normal subject for comparison.

The results corresponding to $c = 0$ revealed the colour deficiency severities of the subjects. Compared with the normal trichromats, the colour deficient people could fail in reporting the correct 'C' direction at small chromaticity difference. From the smallest chromaticity difference at which correct report on the orientation is the majority (3 or 4), we can have a rough idea on the colour deficiency severities of the subjects: While WH and KM have rather mild form of deficiency, CP, SK and KH suffers from much more severe colour deficiency.

Despite the difference in severity, the number of correct recognitions in the four gamut regions being tested increased with compensation level c. This indicated the

Compensation c	0.00				0.25				0.50				0.75			
WHS	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
(N)	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	4	4	4	4	4	4	4	4	4	3	4	4	4	4	4	4
WH	1	3	4	4	4	4	4	4	4	4	4	4	3	4	4	4
(P)	1	2	3	4	4	4	4	4	3	4	4	4	4	4	4	4
	1	2	4	4	4	4	4	4	4	4	4	4	3	4	4	4
	1	4	4	4	2	4	4	4	3	4	4	4	3	4	4	4
KM	2	2	1	3	1	3	4	4	2	3	4	4	2	4	4	4
(P)	0	2	0	1	1	2	1	3	0	3	4	4	4	4	4	4
	0	2	3	3	1	0	2	4	2	2	2	4	0	2	4	4
	2	3	3	2	1	3	1	3	2	2	1	1	1	3	2	2
CP	0	0	0	4	0	4	4	4	4	4	4	4	3	4	4	4
(P)	0	0	0	0	3	4	4	4	4	4	4	4	4	4	4	4
	0	0	4	3	0	0	4	4	0	4	4	4	0	4	4	4
	0	0	0	3	0	0	0	0	4	0	4	4	0	3	4	4

Table 5.3: Results of experiment 5.1 on the effectiveness of GBPC over various gamut regions for subjects using the protanopic confusing set. The results of the subjects at each compensation level are displayed according to the data matrix format defined in Table 5.2.

Compensation c	0.00				0.25				0.50				0.75			
KS	4	4	4	4	4	4	4	4	3	4	4	4	3	4	4	4
(N)	4	4	4	4	2	4	4	4	0	4	4	4	3	4	4	4
	3	4	4	4	3	4	4	4	2	4	4	4	3	4	4	4
	4	4	4	4	4	4	4	4	4	4	4	4	0	4	4	4
SK	0	0	0	0	0	2	1	3	0	4	4	4	4	4	4	4
(D)	0	0	0	0	1	0	3	2	0	3	4	4	0	4	4	4
	0	0	0	3	0	1	4	4	2	4	4	4	2	4	4	4
	0	0	0	0	0	1	0	2	0	2	4	4	0	4	4	4
KH	0	0	0	0	0	0	2	3	0	4	4	4	3	4	4	4
(D)	0	0	0	0	0	2	3	1	0	3	4	4	4	4	4	4
	0	0	0	0	0	4	4	4	4	4	4	4	4	4	4	4
	0	0	0	0	0	1	2	1	0	4	4	4	0	4	4	4

Table 5.4: Results of experiment 5.1 on the effectiveness of GBPC over various gamut regions for subjects using the deuteranopic confusing set. The results of the subjects at each compensation level are displayed according to the data matrix format defined in Table 5.2.

effectiveness of our GBPC compensation algorithm in improving chromatic discrimination over various gamut regions.

One side point to note is the varying compensation levels needed by different people to recover missed chromatic discriminations. Patients like WH who suffer from mild colour deficiency needed a compensation as small as $c = 0.25$ to resume the missed discriminations in our experiment. Patients who suffer from much more severe deficiency, like KH, required as large compensation as $c = 0.75$ to reach the same level. This verifies the relationship between c and deficiency severity discussed in Chapter 4.2.

5.2 Combination of discriminations from the two eyes

Objective

To investigate whether discriminations from the original view and the GBPC-compensated view can be combined.

Stimuli

With the reference chromaticity set at (0.29, 0.43), 16 chromaticity pairs were calculated according to the (chromatic direction, chromaticity difference) combinations shown in Table 5.5. These pairs were aligned on the lines shown in Figure 5.1. The two chromaticities in each pair were used for filling respectively the fields and the backgrounds of four pseudoisochromatic plates, each featuring a 'C' field character in some orientation (up, down, left or right).

	$u'v'$ chromaticity difference			
Chromatic direction	0.03	0.06	0.09	0.12
Protanopic confusion line (Line 1)				
Deuteranopic confusion line (Line 2)				
Direction with slope -0.87 (Line 3)				

Table 5.5: Matrix showing the (chromatic direction, chromaticity difference) combinations used in experiment 5.2

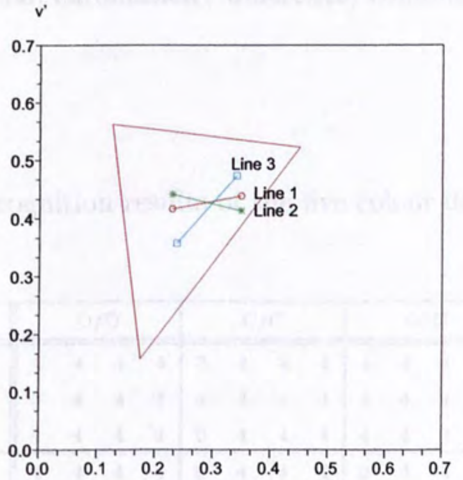


Figure 5.1: The three chromatic directions used in experiment 5.2.

Procedure

Under typical room lighting condition, the 64 pseudoisochromatic plates were presented, four at a time in the form of a 2×2 array at compensation level $c = 0.75$, on a CRT equipped with stereoscopic display capability. Each pseudoisochromatic plate spanned a size of $10cm \times 10cm$ on our 17 inch monitor.

The subject, sitting at 80cm away, viewed the plates through the stereoscope under the following viewing conditions.

- original view to both eyes (O/O)
- compensated view to both eyes (C/C)
- original view to the left eye, compensated view to the right eye (O/C)
- compensated view to the left eye, original view to the right eye (C/O)

The subject was requested to report the 'C' character orientations (up, down, left, right or failed). The number of correct recognitions (0 to 4) corresponding to each (chromatic direction, chromaticity difference) combination was recorded.

Result and analysis

Table 5.6 shows the recognition results of the five colour deficient subjects and two normal subjects.

Viewing condition	O/O				C/C				O/C				C/O			
WHS	4	4	4	4	3	4	4	4	4	4	4	4	4	4	4	4
(N)	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	4	4	4	4	0	4	4	4	4	4	4	4	4	4	4	3
KS	4	4	4	4	0	4	4	4	0	4	4	4	0	4	4	4
(N)	3	4	4	4	3	4	4	4	0	4	4	4	0	4	4	4
	4	4	4	4	0	4	4	4	3	4	4	4	0	4	4	4
WH	1	0	4	4	4	4	4	4	2	4	4	4	4	4	4	4
(P)	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	4	4	4	4	0	1	1	2	2	4	4	4	2	3	2	3
KM	1	1	3	2	2	3	4	4	3	2	4	4	1	2	4	3
(P)	1	4	3	4	3	4	4	4	3	4	4	4	2	4	4	4
	2	4	4	4	2	2	1	4	0	3	2	4	2	2	2	4
CP	0	0	4	3	0	4	4	4	0	3	4	4	0	0	4	4
(P)	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
	4	4	4	4	0	0	0	0	0	0	0	4	0	0	0	4
SK	0	3	4	4	1	2	4	4	0	1	0	4	1	0	3	3
(D)	0	0	0	0	4	4	4	4	3	4	4	4	2	4	4	4
	4	4	4	4	1	0	0	0	0	1	3	4	2	1	4	4
KH	0	3	4	4	0	3	4	4	0	0	0	1	0	0	3	4
(D)	0	0	0	0	4	4	4	4	0	4	4	4	4	4	4	4
	4	4	4	4	0	0	0	0	0	0	4	4	0	0	3	4

Table 5.6: Results of experiment 5.2 on combination of discriminations in the original view and the GBPC-compensated view. The results of each subject at each compensation level are displayed according to the data matrix format defined in Table 5.5.

The results corresponding to the O/O viewing condition revealed the original chromatic discrimination capability of the subjects about the reference chromaticity $(0.29, 0.43)$. While the normal trichromats showed an all-round discrimination at all chromatic direction under test, the colour deficient subjects were not. The protanomalous patients WH, KM and CP had degraded discrimination among the protanopic confusing colours, while the deuteranomalous patients SK and KH confused those deuteranopic confusing colours.

The results corresponding to the C/C viewing condition showed the effect of GBPC about that reference chromaticity. For the colour normal subjects, again their all-round discrimination was not much affected. However, the situation of the colour deficient subjects was different. While discriminations among the originally confusing colours were improved, those among some other colours got degraded. It was especially so with those pairs which were originally on the line with slope -0.87. This is not surprising if we look into how the chromaticity pairs with equal Euclidean distance are transformed under GBPC with $c = 0.75$. As shown in Figure 5.2, those chromaticity pairs originally with slope -0.87 suffer from dramatic drop in their Euclidean distances. The situation is even worse to those protanomalous patients, since the pairs get much closer to the protanopic confusion direction at the same time.

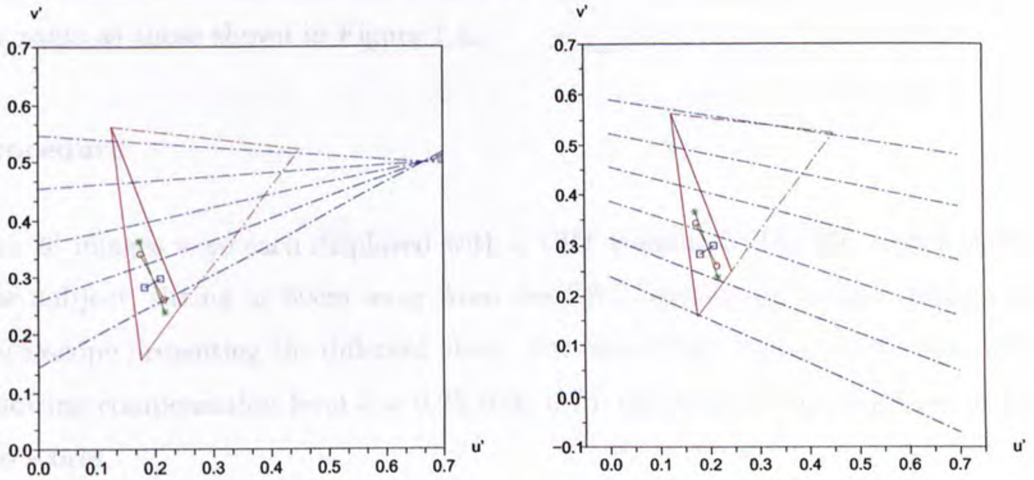


Figure 5.2: The pairs of chromaticities in experiment 5.2 after GBPC $c = 0.75$ over the protanopic confusion lines (left) and the deuteranopic confusion lines (right).

The O/C and C/O viewing conditions resemble the operation of SVDU-MC. The corresponding results indicated how effective the discriminations in the two views are combined at the perceptual level. While the chromatic discrimination of the normal subjects was again unaffected in general, that of colour deficient subjects got much more all-round: While discriminations among originally confusing colours were improved, those which got confused in the compensated view did not show much confusion. Such an outcome was similar to that of tinted filter implementation [14].

5.3 Discrimination improvement and visual comfort

Objective

To investigate if SVDU-MC is beneficial to recovering originally missed discrimination, and comfortable to use in real use cases.

Stimuli

From a standard image database [16], 20 images were randomly selected. They are the same as those shown in Figure 1.1.

Procedure

The 20 images were each displayed with a CRT running SVDU-MC in full screen. The subject, sitting at 80cm away from the CRT, viewed the images through the stereoscope presenting the different views. For each image viewed under each of the following compensation level $c = 0.25, 0.50, 0.75$, the subject was requested to give two scores.

- Overall chromatic discrimination: The scale is from -2 (great loss in image details, through 0 (no change in image details) to +2 (great increase in image details).
- Binocular visual comfort: The scale is from -4 (totally unacceptable even for short-time viewing) to 0 (identical to viewing without compensation).

Result and analysis

While Appendix A show the scores given by the five colour deficient subjects and five normal subjects, here we focus on the interpretation of these results.

Effect of compensation on overall chromatic discrimination

Under this criterion, our principle concern is whether SVDU-MC does recover originally missed discriminations. Tables 5.7 and 5.8 show the tallies on the chromatic discrimination ratings of the colour deficient and normal subjects respectively.

We expect the normal subjects to report no cases of improvement or even degradation. This is related to the palette compression nature of the underlying compensation algorithm GBPC. With the view of one eye switched from the original view to such a less colourful compensated view, the overall colour perception experience can be degraded. This is revealed in Table 5.7, showing the tallies on the chromatic discrimination ratings of the normal subjects. The ratings of most images get degraded on compensation. The smaller the compressed gamut in the compensated view (with higher compensation level c), the poorer the ratings.

In GBPC, the overall colourfulness of the compensated view is sacrificed for displaying the confusing colours with non-confusing entries. So if the colour deficient group exhibits abnormal positive ratings on some images, the usefulness of our scheme may be concluded. This is exactly the result presented in Table 5.8, showing the tallies on the chromatic discrimination ratings of the colour deficient subjects.

Checking how the positive ratings are distributed among the subjects provides another perspective of looking into the result. As shown in Table 5.9, some images (like "frog" and "toucan") were rated positive by almost all colour deficient people. In fact, there are some series of colours that are confused by both L-cone and M-cone defective patients. It was then likely that such a consistency was due to the recovery of discriminations which were lost due to colour deficiency, thus consolidating the

WHS	$c = 0.00$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	0	0	0
+1	0	0	0	0	0	0	0	0
0	20	20	14	14	8	8	8	11
-1	0	0	6	6	9	10	9	7
-2	0	0	0	0	3	2	3	2

KS	$c = 0.00$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	0	0	0
+1	0	0	0	0	0	0	0	0
0	20	20	19	19	7	5	3	3
-1	0	0	1	1	11	12	10	11
-2	0	0	0	0	2	3	7	6

SW	$c = 0.00$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	0	0	0
+1	0	0	0	0	0	0	0	0
0	20	20	16	15	11	11	9	12
-1	0	0	4	5	9	8	11	7
-2	0	0	0	0	0	1	0	1

TW	$c = 0.00$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	0	0	0
+1	0	0	0	1	2	3	0	0
0	20	20	10	6	6	6	5	5
-1	0	0	7	11	10	9	10	11
-2	0	0	2	2	2	2	5	4

DH	$c = 0.00$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	0	0	0
+1	0	0	0	0	0	0	1	1
0	20	20	19	18	15	14	10	7
-1	0	0	1	2	5	6	9	10
-2	0	0	0	0	0	0	5	2

Table 5.7: Tallies on the chromatic discrimination ratings of the colour normal subjects WHS, KS, SW, TW and DH in experiment 5.3.

usefulness of our scheme in real-life image viewing.

WH	c = 0.00		c = 0.25		c = 0.50		c = 0.75	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	0	0	0
+1	0	0	3	3	5	5	6	6
0	20	20	14	14	11	11	9	11
-1	0	0	3	3	4	4	5	3
-2	0	0	0	0	0	0	0	0

KM	c = 0.00		c = 0.25		c = 0.50		c = 0.75	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	2	1	2
+1	0	0	5	10	8	4	5	5
0	20	20	9	4	5	9	3	3
-1	0	0	6	6	7	5	11	10
-2	0	0	0	0	0	0	0	0

CP	c = 0.00		c = 0.25		c = 0.50		c = 0.75	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	1	0	4	0	0
+1	0	0	5	4	7	3	7	9
0	20	20	11	11	0	0	1	0
-1	0	0	4	4	13	13	12	11
-2	0	0	0	0	0	0	0	0

SK	c = 0.00		c = 0.25		c = 0.50		c = 0.75	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	0	0	0	1	1	1
+1	0	0	1	1	1	2	1	1
0	20	20	18	17	12	10	10	10
-1	0	0	1	2	7	7	4	4
-2	0	0	0	0	0	0	4	4

KH	c = 0.00		c = 0.25		c = 0.50		c = 0.75	
Rating	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
+2	0	0	1	2	1	4	0	4
+1	0	0	4	5	9	5	9	9
0	20	20	15	13	10	11	11	7
-1	0	0	0	0	0	0	0	0
-2	0	0	0	0	0	0	0	0

Table 5.8: Tallies on the chromatic discrimination ratings of the colour deficient subjects WH, KM, CP, SK and KH in experiment 5.3.

Effect of compensation on visual comfort

As shown in Experiments 5.1 and 5.2, the severe colour deficient people can demand a compensation level as high as $c = 0.75$ for decent chromatic discrimination recovery.

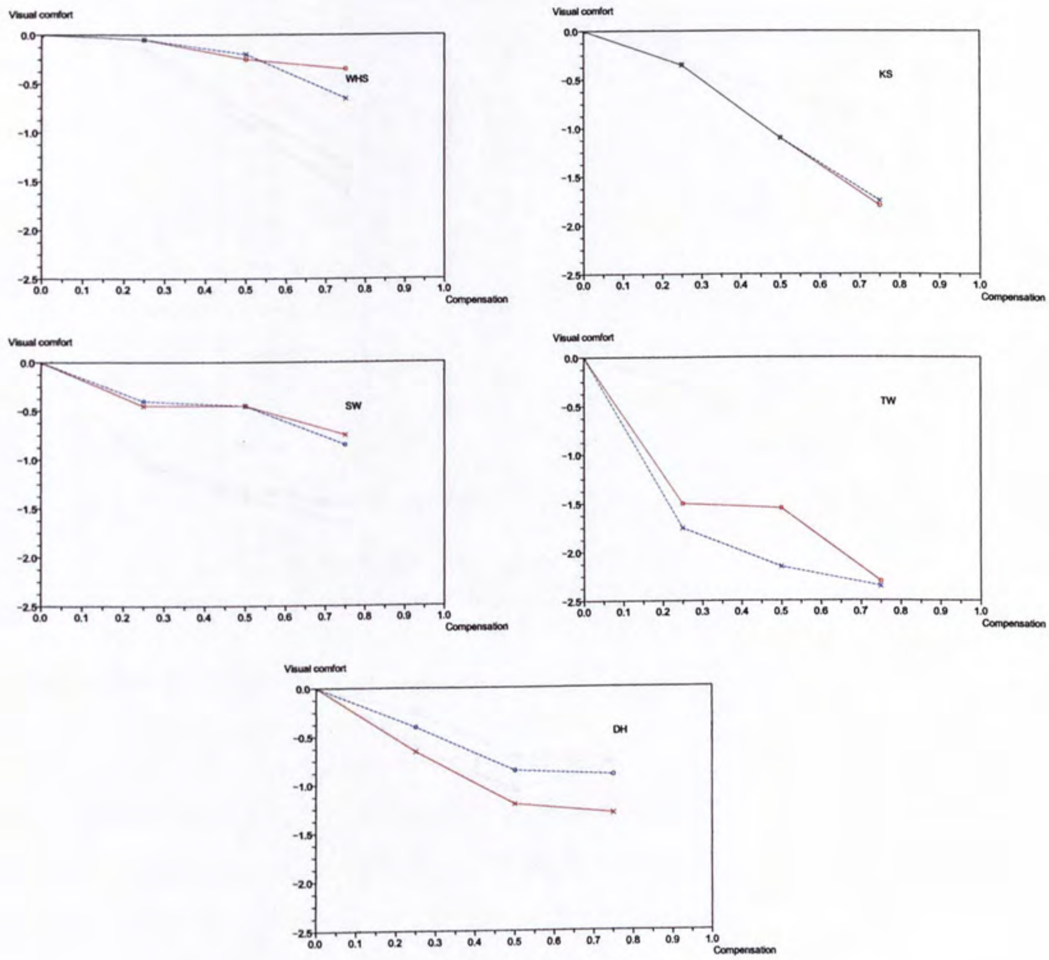
Subject	WHS	KS	SW	TW	DH	WH	KM	CP	SK	KH
Deficiency type	N	N	N	N	N	P	P	D	D	D
Compensation level c	N/A	N/A	N/A	0.50	0.75	0.75	0.25	0.75	0.50	0.75
Viewing condition	N/A	N/A	N/A	C/O	Both	Both	C/O	C/O	C/O	C/O
Dominant eye	L	L	R	L	R	L	L	L	L	L
anhinga										$\surd$
athens						$\surd$				$\surd$
baboon							$\surd$	$\surd$		$\surd$
barnfall						$\surd$			$\surd$	$\surd$
bird										
elephant						$\surd$				$\surd$
frog						$\surd$	$\surd$	$\surd$	$\surd$	$\surd$
avion							$\surd$			
lena										
london				$\surd$			$\surd$			$\surd$
lostlake							$\surd$			$\surd$
malight										
mare					$\surd$			$\surd$		$\surd$
oldmill							$\surd$			
peppers							$\surd$	$\surd$		$\surd$
portofino							$\surd$	$\surd$		
sailboad						$\surd$	$\surd$	$\surd$		$\surd$
stagcach								$\surd$		$\surd$
stonehse										
toucan						$\surd$	$\surd$	$\surd$	$\surd$	$\surd$

Table 5.9: Images granted positive chromatic discrimination ratings in experiment 5.3. They are indicated with a '$\surd$' symbol in the corresponding box. The statistics of each subject are taken from the particular (compensation level c, viewing condition) combination which gives the largest number of positive ratings (+1 and +2). For the "dominant eye" row, 'L' means left and 'R' means right.

However, the setting should not be welcomed if the associated visual comfort is too high.

Under GBPC, colours close to the red-blue boundary of the VDU gamut suffer from the largest change, while those close to the green-blue boundary act in the opposite manner. So images making more extensive use of red (like "baboon", "lena" and "pepper") produce larger binocular dissimilarity and hence offer higher visual discomfort than other images as shown in Table 5.10. So a fair metric for this criterion is to see if the average comfort is of an acceptable level. Tables 5.3 and 5.4 show these ratings.

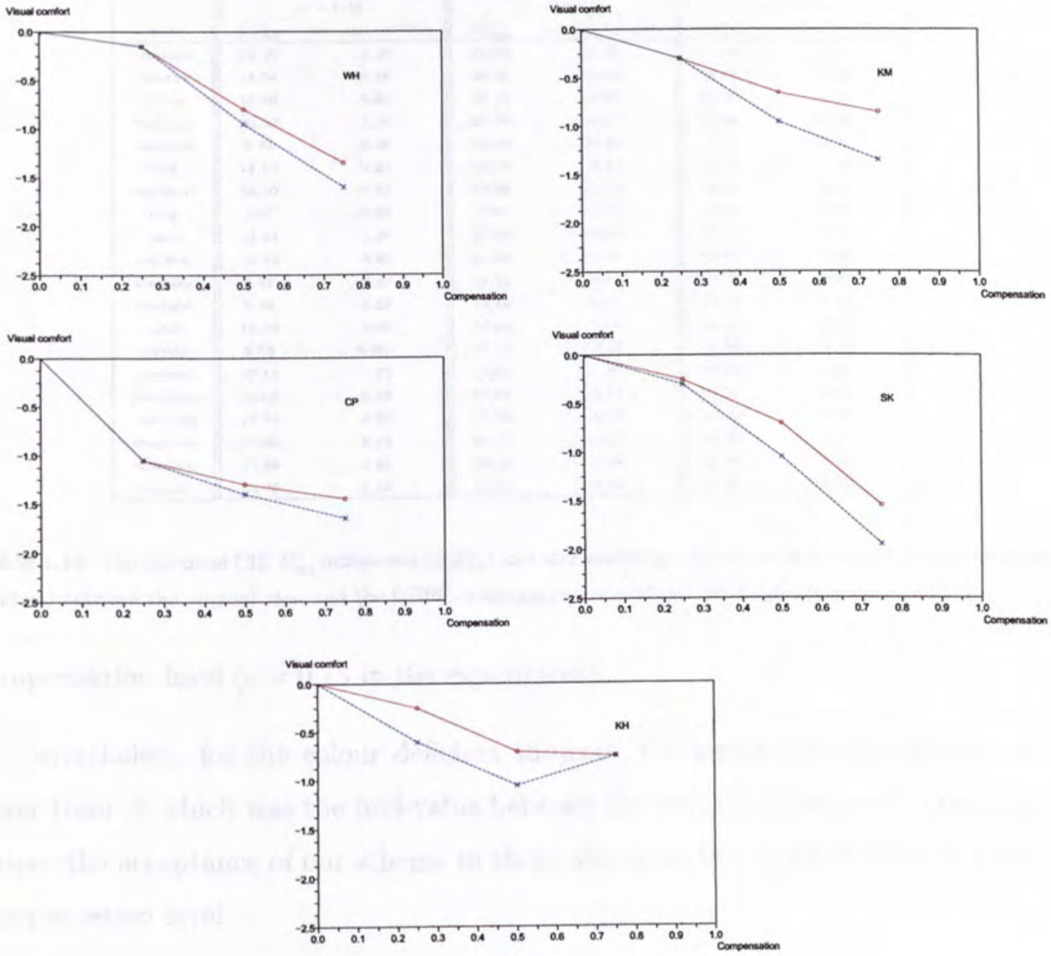
In general, the higher the compensation level c, the lower the visual comfort



	c = 0.00		c = 0.25		c = 0.50		c = 0.75	
Subject	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
WHS (N)	0.00	0.00	-0.05	-0.05	-0.25	-0.20	-0.35	-0.65
KS (N)	0.00	0.00	-0.35	-0.35	-1.10	-1.10	-1.80	-1.75
SW (N)	0.00	0.00	-0.40	-0.45	-0.45	-0.45	-0.85	-0.75
TW (N)	0.00	0.00	-1.50	-1.75	-1.55	-2.15	-2.30	-2.35
DH (N)	0.00	0.00	-0.40	-0.65	-0.85	-1.20	-0.90	-1.30

Figure 5.3: Average visual comfort of the normal subjects on the test images viewed under SVDU-MC at different GBPC c values in experiment 5.3. The data points corresponding to the O/C viewing condition and the C/O viewing condition are marked with 'o' and 'x' respectively. The points are joined with red solid line if they correspond to the compensated view presented to the non-dominant eye. The points are joined with blue dash line if they correspond to the compensated view presented to the dominant eye.

rating. Intuitively, higher compensation is associated with larger colour change on the compensated image, and hence larger binocular dissimilarity. In fact, as shown



Subject	c = 0.00 O/C	c = 0.00 C/O	c = 0.25 O/C	c = 0.25 C/O	c = 0.50 O/C	c = 0.50 C/O	c = 0.75 O/C	c = 0.75 C/O
WH (P)	0.00	0.00	-0.15	-0.15	-0.80	-0.95	-1.35	-1.60
KM (P)	0.00	0.00	-0.30	-0.30	-0.65	-0.95	-0.85	-1.35
CP (P)	0.00	0.00	-1.05	-1.05	-1.30	-1.40	-1.45	-1.65
SK (D)	0.00	0.00	-0.25	-0.30	-0.70	-1.05	-1.55	-1.95
KH (D)	0.00	0.00	-0.25	-0.60	-0.70	-1.00	-0.75	-0.75

Figure 5.4: Average visual comfort of the colour deficient subjects on the test images viewed under SVDU-MC at different GBPC c values in experiment 5.3. The data points corresponding to the O/C viewing condition and the C/O viewing condition are marked with 'o' and 'x' respectively. The points are joined with red solid line if they correspond to the compensated view presented to the non-dominant eye. The points are joined with blue dash line if they correspond to the compensated view presented to the dominant eye.

in Table 5.10, the per-pixel E^*_{uv} difference increases as c gets larger for all images. As a result, the visual comfort averages of all subjects are the lowest at the highest

Image	c = 0.25		c = 0.50		c = 0.75	
	ΔE^*_{uv}	average comfort	ΔE^*_{uv}	average comfort	ΔE^*_{uv}	average comfort
anhinga	12.49	-0.40	24.72	-0.65	37.05	-1.50
athens	14.74	-0.35	29.30	-0.35	44.01	-0.85
avion	19.96	-0.30	40.17	-0.65	62.56	-1.15
baboon	20.00	-1.20	39.70	-2.05	59.86	-2.75
barnfall	9.29	-0.35	18.52	-0.65	27.75	-1.20
bird	11.58	-0.25	23.01	-0.55	34.53	-1.15
elephant	16.10	-0.45	32.00	-0.65	48.31	-1.20
frog	1.51	-0.30	2.90	-0.70	4.33	-0.80
lena	34.44	-1.35	68.34	-2.05	103.15	-2.15
london	13.55	-0.40	26.49	-0.40	39.65	-0.80
lostlake	9.48	-0.40	18.62	-0.45	27.72	-0.70
malight	9.48	-0.30	19.10	-0.75	29.06	-1.10
mare	15.55	-0.95	30.94	-2.00	46.41	-2.40
oldmill	9.02	0.00	17.83	-0.45	26.86	-0.55
peppers	27.11	-1.20	53.80	-1.95	80.96	-2.40
portofino	14.03	-0.35	27.91	-0.75	41.96	-0.95
sailboad	17.59	-0.30	35.00	-0.65	53.22	-0.80
stagcach	10.66	-0.75	21.23	-1.20	31.92	-1.50
stonehse	11.83	-0.25	23.49	-0.60	35.34	-0.53
toucan	13.79	-0.55	27.27	-1.30	41.06	-1.40

Table 5.10: The per-pixel CIE E^*_{uv} differences (ΔE^*_{uv}) and corresponding subject-average comfort ratings (average comfort) between the original view and the GBPC-compensated view of the test images in experiment 5.3.

compensation level ($c = 0.75$ in the experiment).

Nevertheless, for the colour deficient subjects, the lowest averages did not get lower than -2 which was the mid-value between the worst and the best. This guarantees the acceptance of our scheme to those who need to run SVDU-MC at a high compensation level.

An interesting phenomenon to note is the different binocular dissimilarity tolerance of different subjects. There exists an inter-person variation on the comfort rating for the same image at the same compensation level. The subjective nature of this quantity suggests a need to preserve compensation flexibility from the perspective of visual comfort. Those who do not prefer too high a visual comfort can have a lower compensation level, at which the overall binocular dissimilarity and hence visual comfort is smaller.

Effect of eye dominance on visual comfort and overall chromatic discrimination

Our formulation of SVDU-MC does not specify the eye to which the compensated view is presented. To investigate whether this factor affects chromatic discrimination and visual comfort, paired t-tests are carried out with the following null hypothesis H_0 and alternative hypothesis H_1:

$$H_0 : v_{O/C_i} = v_{C/O_i}$$

$$H_1 : v_{O/C_i} \neq v_{C/O_i}$$

where v_{O/C_i} and v_{C/O_i} refer to the ratings (chromatic discrimination or visual comfort) corresponding to an image i under the O/C and C/O viewing condition respectively. Table 5.11 shows the t statistics so calculated.

	$c = 0.00$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Subject	details	comfort	details	comfort	details	comfort	details	comfort
WHS (N)	N/A	N/A	N/A	N/A	0.57	1.00	2.18	-2.85
KS (N)	N/A	N/A	N/A	N/A	-1.83	N/A	0.57	0.57
SW (N)	N/A	N/A	-1.00	-1.00	-0.57	N/A	1.00	1.00
TW (N)	N/A	N/A	-1.45	-1.75	1.00	-4.49	0.57	-0.29
DH (N)	N/A	N/A	-1.00	-2.52	-1.00	-3.20	-2.52	-3.56
WH (P)	N/A	N/A	N/A	N/A	N/A	-1.83	1.45	-2.52
KM (P)	N/A	N/A	1.56	0.00	0.70	-2.35	1.14	-3.68
CP (P)	N/A	N/A	1.00	N/A	2.18	-1.45	1.37	-2.18
SK (D)	N/A	N/A	-1.00	-1.00	0.90	-3.20	N/A	-3.56
KH (D)	N/A	N/A	1.83	-3.20	1.00	-2.35	2.63	N/A

Table 5.11: Paired t-statistics comparing the effect of presenting the GBPC-compensated view to different eyes on the chromatic discrimination (details) and visual comfort (comfort) ratings in experiment 5.3. N/A refers to the situation that no difference exists between the ratings for all images. The corresponding t value for 95% confidence (two-tailed) ranges between -2.093 and 2.093.

As c increased, more and more subjects had the t value fall into the null hypothesis rejection level (95% confidence, two-tailed) when visual comfort was in concern. Among the 10 subjects, the people counts were 2 for $c = 0.25$, 5 for $c = 0.50$ and 6 for $c = 0.75$. The eye to which the compensated view is presented seems affecting visual comfort at high compensation levels.

X-Chrom and ChromaGen suggest fitting the tinted lens which generates the compensated view on the non-dominant eye [10, 33]. This way of presenting the compensated view matches our observation from the raw data (Figures 5.4 and 5.3): Feeding the compensated view to the dominant eye usually yielded higher visual discomfort than the other way round. Only DH presented an exception.

In contrast, the proportion of subjects falling into the null hypothesis rejection level was much smaller when chromatic discrimination was concerned instead. Among the 10 subjects, the people counts were 0 for $c = 0.25$, 1 for $c = 0.50$ and 3 for $c = 0.75$. Looking into the raw data of these cases, we find that in images showing positive ratings, showing the compensated view to the dominant eye yielded much higher positive rating than the alternative presentation approach. In fact, as shown in Table 5.9, the viewing conditions offering the largest number of positively rated images were usually the one with the compensated view presented to the dominant eye.

So our suggestion is as follows: To start with, adjust c with the compensated view shown to the non-dominant eye. After the compensation level has been determined, the viewer can choose to have the compensated view presented to the dominant eye for occasional better chromatic discrimination, at the expense of higher visual discomfort.

Chapter 6

Conclusion and Future Works

Conclusion

We have completed our discussion on SVDU-MC as a colour vision improvement scheme for use with VDUs. It works with protanopic, protanomalous, deuteranopic and deuteranomalous patients who comprise the majority of the colour-blind population. It aims at recovering those chromatic discriminations that are missed as a result of colour deficiency.

SVDU-MC uses monocular compensation presentation strategy. We have emphasized its necessity in preserving original discriminations which may be lost in compensation. To reduce binocular dissimilarity especially in the luminance domain, the underlying compensation algorithm needs careful design. We have explained how our compensation algorithm named gamut-based palette compression (GBPC) can maximize the compensation under this constraint.

Experiments with both artificial and real-life images have verified our design. In particular, the occasional positive chromatic discrimination rating reported by the colour deficient subjects demonstrates the value of our system.

Future works

The results from the experiments in Chapter 5 not only verify the intended effect of SVDU-MC, but also inspire us with a few directions that are worth further development. They are listed as follows.

Refinement of compensation algorithm

Under our current implementation, all sets of confusing colours, despite their locations in the gamut, undergo similar tilting and hence compensation. These compensation would have been equally worthwhile if the chromatic discrimination had been degraded by similar extent over the gamut. However, as shown in the results corresponding to GBPC $c = 0$ setting of experiment 5.1, the degradation can be different at different gamut regions. If compensation in these less degraded regions can be lowered, unnecessary binocular dissimilarity can be reduced, which in turn lowers the associated undesirable visual discomfort.

It would be especially helpful if the subsequent design can reduce the colour change over the red gamut region. Under the current GBPC scheme, those red colours suffer from the largest change among all colours. In fact, images that have the largest binocular difference are those with the extensive use of red.

Studying the chromatic discrimination degradation at different gamut regions, and then introducing suitable non-linearity to GBPC may be a good starting point.

Calibration procedure

Our algorithm features a parameter c which corresponds to the compensation level. One may end up with a linear adjustment control. The viewer can change the c setting freely at any time, probably according to the current viewing content. However, this can be troublesome. What is more desirable is a c setting that works nicely in all situations for a particular viewer.

Experiment 5.3 is in fact our attempt to tackle this problem. It is based on the two important qualities that a person would consider when selecting the compensation level - visual comfort and overall chromatic discrimination. Results show that while the former exhibit linear relation with c, the latter is much more complicated. Higher compensation can no doubt improve chromatic discrimination in regions with confusing colours, as shown in Experiments 5.1. But at the same time, discriminations in some other originally non-confusing regions can get worse, as shown in Experiment 5.2. Which dominates the overall chromatic discrimination scoring depends on the not only the objective criterion of relative extensiveness of these regions, but also the subjective criterion of relative significance of these changes to the viewers. This may explain why we see the scores in Experiment 5.3 varied from images to images, and also from person to person. Furthermore, the effect of adaptation is yet to be investigated. The responses of the subjects may change after they get used to the SVDU-MC viewing environment.

To further explore this issue, one needs to find out the relative importance of these factors. Only with this information can the calibration tasks be designed. Experiments with a large number of colour deficient subjects would be useful.

Final Words

With the rapid advance of multimedia and telecommunication technologies, the human visual world is now filled with images reproduced with VDUs. SVDU-MC is exactly the tool targeting at this kind of colour image reproduction media. The scheme requires the VDUs to be equipped with stereoscopic display capability, which can now be easily fulfilled with the widely available and affordable stereoscope, or even autostereoscopy technology using no bulky shutter glasses. We expect that SVDU-MC can become an effective, useful, affordable and handy colour vision aid for the colour-blind.

Appendix A

Raw Data of Experiment 5.3

This chapter presents the raw data collected in experiement 5.3. They are the bases of the analyses presented in Section 5.3 of the main text.

Scores reported by colour normal subjects

Criteria	Overall chromatic discrimination						Visual comfort					
	$c = 0.25$		$c = 0.50$		$c = 0.75$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	-1	-1	-2	-2	0	0	0	0	-2	-3
athens	0	0	0	0	0	0	0	0	0	0	0	0
avion	0	0	0	0	0	0	0	0	0	0	0	-1
baboon	-1	-1	-1	-2	0	0	-1	-1	-1	-1	-1	-2
branfall	0	0	-1	-1	0	0	0	0	0	0	0	-1
bird	0	0	0	0	0	0	0	0	0	0	0	0
elephant	0	0	0	0	0	0	0	0	0	0	-1	-1
frog	-1	-1	-1	-1	-2	-1	0	0	0	0	0	0
lena	-1	-1	-2	-1	-1	-1	0	0	-1	-1	-1	-1
london	0	0	0	0	0	0	0	0	0	0	0	0
lostlake	0	0	-1	-1	-1	-1	0	0	0	0	0	0
malight	0	0	0	0	-1	-1	0	0	0	0	0	0
mare	-1	-1	-1	-1	-1	-1	0	0	-1	-1	-1	-2
oldmill	0	0	0	0	0	0	0	0	0	0	0	-1
peppers	-1	-1	-2	-2	-2	-2	0	0	-1	-1	-1	-1
portofino	0	0	-1	-1	-1	-1	0	0	0	0	0	0
sailboad	0	0	0	0	-1	0	0	0	0	0	0	0
stagcach	0	0	-1	-1	-1	-1	0	0	0	0	0	0
stonehse	0	0	-1	-1	-1	0	0	0	0	0	0	0
toucan	-1	-1	-2	-1	-1	0	0	0	-1	0	0	0

Table A.1: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject WHS

Criteria	Overall chromatic discrimination						Visual comfort					
	$c = 0.25$		$c = 0.50$		$c = 0.75$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
athens	0	0	-2	-2	-2	-2	0	0	0	0	-1	-1
avion	0	0	0	0	-1	-1	0	0	-1	-1	-1	-1
baboon	0	0	0	-1	-1	-1	-1	-1	-2	-2	-3	-3
branfall	0	0	0	0	0	0	-1	-1	-1	-1	-1	-1
bird	0	0	0	0	-2	-2	0	0	-1	-1	-2	-2
elephant	0	0	0	-1	-1	-1	-1	-1	-1	-1	-2	-3
frog	0	0	0	0	-1	-1	0	0	-1	-1	-1	-1
lena	0	0	0	0	-2	-2	-1	-1	-2	-2	-4	-3
london	0	0	-1	-1	-1	-1	0	0	-1	-1	-1	-1
lostlake	0	0	-1	-1	0	0	0	0	0	0	-1	-1
malight	0	0	-1	-1	-2	-1	0	0	0	0	-1	-1
mare	0	0	-1	-2	-2	-2	-1	-1	-2	-2	-4	-3
oldmill	0	0	-1	-1	-2	-1	0	0	-1	-1	-1	-1
peppers	0	0	-1	-1	-1	-1	-1	-1	-2	-2	-3	-3
portofino	0	0	-2	-2	-2	-2	0	0	-1	-1	-3	-3
sailboad	0	0	-1	-1	-1	-1	0	0	-1	-1	-1	-1
stagcach	0	0	-1	-1	-1	-2	0	0	-1	-1	-2	-2
stonehse	0	0	-1	-1	0	0	0	0	-1	-1	-1	-1
toucan	-1	-1	-1	-1	-1	-1	0	0	-2	-2	-2	-2

Table A.2: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject KS

Criteria	Overall chromatic discrimination						Visual comfort					
	$c = 0.25$		$c = 0.50$		$c = 0.75$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	0	0	0	0	0	0	0	0	-1	-1
athens	0	0	0	0	0	0	0	0	0	0	-1	-1
avion	0	0	0	0	0	0	0	0	0	0	0	0
baboon	0	0	-1	-2	-1	-1	-1	-2	-2	-2	-2	-2
branfall	-1	-1	-1	-1	0	0	0	0	-1	-1	-1	-1
bird	0	0	0	0	-1	0	0	0	0	0	-1	-1
elephant	0	0	0	0	0	0	0	0	0	0	0	0
frog	-1	-1	-1	-1	-1	-1	0	0	0	0	0	0
lena	0	0	0	0	0	0	-1	-1	-1	-1	-1	0
london	0	0	0	0	0	0	0	0	0	0	-1	0
lostlake	0	0	0	0	-1	-1	-1	-1	0	0	-1	-1
malight	0	-1	-1	-1	0	0	0	0	-1	-1	0	-1
mare	-1	-1	-1	-1	-1	-1	-2	-2	-2	-2	-2	-2
oldmill	0	0	-1	-1	-1	-1	0	0	0	0	0	0
peppers	-1	-1	-1	0	-1	0	-1	-1	0	0	-2	-1
portofino	0	0	0	0	-1	-1	0	0	0	0	0	0
sailboad	0	0	-1	-1	-1	0	0	0	0	0	0	0
stagcach	0	0	0	-1	-1	-2	-1	-1	-1	-1	-2	-2
stonehse	0	0	-1	-1	-1	-1	0	0	0	0	0	0
toucan	0	0	0	0	0	0	-1	-1	-1	-1	-2	-2

Table A.3: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject SW

Criteria	Overall chromatic discrimination						Visual comfort					
	$c = 0.25$		$c = 0.50$		$c = 0.75$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	0	0	-1	-1	-1	-1	-2	-2	-1	-2
athens	-1	-1	0	0	-1	-1	-1	-1	0	-1	-3	-2
avion	0	-1	-1	-1	-1	0	-1	-1	-1	-1	-3	-3
baboon	0	0	0	0	-1	-1	-2	-3	-3	-3	-4	-4
branfall	-1	-1	-2	-2	-1	-1	-1	-2	-1	-2	-2	-3
bird	0	-1	-1	-1	-1	-1	-1	-1	0	-1	-2	-1
elephant	0	1	1	1	0	0	-1	-1	-1	-2	-2	-1
frog	-1	-1	0	0	0	0	-1	-2	-1	-1	-2	-2
lena	-1	-1	0	0	-2	-2	-3	-3	-3	-4	-4	-4
london	-1	-1	-1	1	-1	-1	-1	-2	-1	-1	-2	-2
lostlake	0	0	-1	-1	-2	-2	-2	-2	-1	-1	-1	-1
malight	-2	-2	-1	-1	-1	-1	-1	-2	0	0	-1	-2
mare	1	-1	1	1	-2	-2	-2	-1	-3	-4	-4	-3
oldmill	0	-1	0	0	-2	-1	0	0	-1	-1	-1	-1
peppers	0	0	-1	-1	0	0	-3	-4	-3	-4	-3	-2
portofino	-1	-1	-1	-1	-1	-1	-2	-1	-2	-2	-1	-1
sailboad	-1	-1	-1	-1	0	-1	-1	-1	-1	-2	-3	-3
stagcach	-2	-2	-2	-2	-2	-2	-3	-3	-3	-4	-3	-4
stonehse	0	0	-1	-1	0	0	-1	-1	-1	-3	-1	-2
toucan	0	0	-1	-1	-1	-1	-2	-3	-3	-4	-3	-4

Table A.4: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject TW

Criteria	Overall chromatic discrimination						Visual comfort					
	$c = 0.25$		$c = 0.50$		$c = 0.75$		$c = 0.25$		$c = 0.50$		$c = 0.75$	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	-1	-1	0	-1	-1	-1	-1	-1	-1	-2
athens	0	0	0	0	0	0	0	0	-1	-1	0	-1
avion	0	0	-1	-1	-1	-2	0	0	-1	-1	-1	-1
baboon	0	0	0	0	-1	-1	-1	-1	-1	-2	-2	-2
branfall	0	0	0	0	0	0	0	0	-1	-1	-1	-1
bird	0	0	0	0	-1	-1	0	0	-1	-1	0	-1
elephant	-1	-1	-1	-1	-1	-2	-1	-1	-1	-1	-1	-1
frog	0	0	0	0	0	0	0	0	-1	-1	-1	-1
lena	0	0	0	0	-1	-1	-1	-2	-1	-2	-1	-2
london	0	0	0	0	0	0	0	0	0	0	-1	-1
lostlake	0	0	0	0	-1	-1	0	0	0	0	-1	-2
malight	0	-1	0	-1	0	-1	0	0	-1	-2	-1	-1
mare	0	0	0	0	1	1	-1	-1	-1	-2	-1	-2
oldmill	0	0	0	0	0	-1	0	0	0	0	0	0
peppers	0	0	0	0	0	0	-1	-2	-1	-2	-1	-1
portofino	0	0	-1	-1	-1	-1	0	-1	-1	-2	-1	-1
sailboad	0	0	0	0	-1	-1	0	0	-1	-1	-1	-1
stagcach	0	0	0	0	0	0	-1	-2	-1	-2	-1	-2
stonehse	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
toucan	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2

Table A.5: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject DH

Scores reported by colour deficient subjects

Criteria	Overall chromatic discrimination						Visual comfort					
	c = 0.25		c = 0.50		c = 0.75		c = 0.25		c = 0.50		c = 0.75	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	0	0	0	0	0	0	0	0	0	-1
athens	0	0	0	0	1	1	0	0	0	0	0	0
avion	0	0	0	0	-1	0	0	0	0	0	-1	-1
baboon	0	0	0	0	0	0	-1	-1	-2	-2	-4	-4
branfall	1	1	1	1	1	1	0	0	0	0	-1	-1
bird	0	0	0	0	0	0	0	0	0	0	-1	-2
elephant	0	0	1	1	1	1	0	0	0	0	-1	-2
frog	1	1	1	1	1	1	0	0	-1	-1	-2	-2
lena	0	0	0	0	0	0	-1	-1	-3	-4	-4	-4
london	0	0	0	0	0	0	0	0	0	0	0	0
lostlake	0	0	-1	-1	-1	0	0	0	-1	-1	0	0
malight	-1	-1	0	0	0	0	0	0	0	-1	-2	-2
mare	-1	-1	-1	-1	-1	-1	0	0	-3	-3	-3	-3
oldmill	0	0	0	0	0	0	0	0	0	0	0	0
peppers	0	0	0	0	0	0	-1	-1	-4	-4	-4	-4
portofino	0	0	-1	-1	-1	-1	0	0	0	0	-1	-1
sailboad	0	0	1	1	1	1	0	0	0	0	0	0
stagcach	-1	-1	-1	-1	-1	-1	0	0	-1	-1	-2	-3
stonehse	0	0	0	0	0	0	0	0	0	0	0	0
toucan	1	1	1	1	1	1	0	0	-1	-2	-1	-2

Table A.6: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject WH

Criteria	Overall chromatic discrimination						Visual comfort					
	c = 0.25		c = 0.50		c = 0.75		c = 0.25		c = 0.50		c = 0.75	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	-1	-1	-1	-1	-1	-1	0	0	0	0	-1	-2
athens	0	-1	-1	0	-1	-1	0	0	0	0	0	-1
avion	1	1	0	0	0	1	0	-1	-1	-2	-1	-1
baboon	0	-1	1	1	1	1	-1	-1	-2	-3	-2	-3
barnfall	-1	-1	-1	-1	-1	0	0	0	0	0	0	0
bird	-1	0	-1	0	-1	-1	0	0	0	0	-1	-1
elephant	0	-1	-1	0	-1	-1	0	0	0	0	0	-1
frog	0	1	1	2	2	2	0	0	0	0	0	0
lena	-1	-1	0	-1	-1	-1	-1	-2	-2	-3	-3	-4
london	1	1	-1	-1	-1	-1	0	0	0	0	0	-1
lostlake	1	1	0	0	0	-1	0	0	-1	-1	-1	0
malight	0	-1	1	0	-1	-1	0	0	-1	-1	-1	-2
mare	0	0	1	1	1	0	-1	-1	-1	-2	-2	-3
oldmill	0	1	-1	-1	-1	-1	0	0	0	-1	0	-1
peppers	1	1	1	1	1	1	-1	-1	-2	-3	-3	-4
portofino	0	1	1	1	-1	0	-1	0	-1	0	-1	-1
sailboad	0	1	1	0	1	1	-1	0	-1	-1	-1	-1
stagcach	-1	0	0	0	-1	-1	0	0	-1	-1	0	-1
stonehse	-1	0	0	0	0	1	0	0	0	-1	0	0
toucan	1	1	1	2	1	2	0	0	0	0	0	0

Table A.7: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject KM

Criteria	Overall chromatic discrimination						Visual comfort					
	c = 0.25		c = 0.50		c = 0.75		c = 0.25		c = 0.50		c = 0.75	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2
athens	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
avion	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
baboon	1	1	1	1	-1	1	-2	-2	-3	-3	-4	-4
barnfall	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2
bird	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
elephant	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
frog	1	1	1	2	1	1	-1	-1	-2	-2	-1	-1
lena	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2	-2	-3
london	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
lostlake	0	0	1	1	0	1	-1	-1	-1	-1	-1	-1
malight	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
mare	1	1	1	2	1	1	-2	-2	-2	-2	-2	-2
oldmill	0	0	-1	-1	-1	-1	0	0	-1	-1	-1	-1
peppers	1	2	1	2	1	1	-2	-2	-2	-3	-4	-4
portofino	0	0	-1	-1	1	1	0	0	-1	-1	-1	-1
sailboad	0	0	-1	-1	1	1	-1	-1	-1	-1	-1	-1
stagcach	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1
stonehse	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
toucan	1	1	1	2	1	1	-1	-1	-2	-2	-2	-3

Table A.8: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC *c* settings for subject CP

Criteria	Overall chromatic discrimination						Visual comfort					
	c = 0.25		c = 0.50		c = 0.75		c = 0.25		c = 0.50		c = 0.75	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	-1	-1	-2	-2	0	0	-1	-2	-3	-3
athens	0	-1	-1	-1	-2	-2	0	0	0	0	-1	-1
avion	0	0	-1	-1	-1	-1	0	0	0	0	-2	-2
baboon	0	0	0	0	-1	-1	-1	-1	-2	-3	-3	-4
barnfall	1	1	1	2	2	2	0	0	0	0	-2	-3
bird	0	0	-1	-1	0	0	0	-1	-1	-1	-2	-2
elephant	0	0	-1	-1	-2	-2	0	0	0	0	0	0
frog	0	0	1	1	1	1	0	0	-1	-1	-1	-1
lena	0	0	0	0	0	0	-1	-1	-2	-3	-3	-4
london	0	0	0	0	0	0	0	0	0	0	-1	-1
lostlake	0	0	0	0	0	0	0	0	0	-1	-1	-1
malight	0	0	-1	-1	-1	-1	0	0	0	0	-1	-2
mare	0	0	0	0	0	0	-1	-1	-2	-3	-3	-4
oldmill	-1	-1	-1	-1	-2	-2	0	0	0	0	0	-1
peppers	0	0	0	0	0	0	-1	-1	-2	-3	-3	-4
portofino	0	0	0	0	-1	-1	-1	-1	0	0	0	-1
sailboad	0	0	0	0	0	0	0	0	0	0	-1	-1
stagcach	0	0	0	0	0	0	0	0	-1	-1	-1	-1
stonehse	0	0	-1	-1	-2	-2	0	0	0	0	0	0
toucan	0	0	0	1	0	0	0	0	-1	-2	-1	-1

Table A.9: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC *c* settings for subject SK

Criteria	Overall chromatic discrimination						Visual comfort					
	c = 0.25		c = 0.50		c = 0.75		c = 0.25		c = 0.50		c = 0.75	
Image	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O	O/C	C/O
anhinga	0	0	1	1	0	1	0	0	0	-1	-1	-1
athens	0	0	0	0	0	1	-1	-2	-1	-1	-1	-1
avion	0	0	0	0	0	0	0	-1	-1	-1	-1	-1
baboon	1	1	1	1	1	1	0	0	-1	-1	-1	-1
barnfall	2	2	2	2	0	2	0	0	-1	-1	-1	-1
bird	0	0	0	0	0	0	0	0	-1	-1	-1	-1
elephant	0	0	0	0	1	1	0	-1	-1	-1	-1	-1
frog	1	1	1	2	1	2	0	0	0	0	0	0
lena	0	0	0	0	0	0	-2	-3	-1	-2	-1	-1
london	0	1	1	2	1	1	-1	-2	-1	-1	-1	-1
lostlake	0	0	1	0	1	1	0	0	0	0	0	0
malight	0	0	0	0	0	0	0	-1	-2	-3	-1	-1
mare	0	1	1	1	1	1	0	0	-1	-1	-1	-1
oldmill	0	0	0	0	0	0	0	0	-1	-1	-1	-1
peppers	1	2	1	2	1	2	0	0	0	0	0	0
portofino	0	0	0	0	0	0	0	0	-1	-2	-1	-1
sailboad	0	0	0	0	0	2	0	-1	-1	-1	0	0
stagcach	0	0	1	1	1	1	-1	-1	0	-2	-1	-1
stonehse	0	0	0	0	0	0	0	0	0	-1	-1	-1
toucan	1	1	1	1	1	1	0	0	0	0	0	0

Table A.10: Results of experiment 5.3 on the overall chromatic discrimination and visual comfort under different SVDU-MC c settings for subject KH

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